Neural Network Adaptive Control for Nonlinear Nonnegative Dynamical Systems

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***Abstract*—Nonnegative and compartmental dynamical system models are derived from mass and energy balance considerations that involve dynamic states whose values are nonnegative. These models are widespread in engineering and life sciences and typically involve the exchange of nonnegative quantities between subsystems or compartments wherein each compartment is assumed to be kinetically homogeneous. In this paper, we develop a full-state feedback neural adaptive control framework for adaptive set-point regulation of nonlinear uncertain nonnegative and compartmental systems. The proposed framework is Lyapunov-based and guarantees ultimate boundedness of the error signals corresponding to the physical system states and the neural network weighting gains. In addition, the neural adaptive controller guarantees that the physical system states remain in the nonnegative orthant of the state–space for nonnegative initial conditions.**

***Index Terms*—Adaptive control, neural networks, nonlinear compartmental systems, nonlinear nonnegative systems, nonnegative control, set-point regulation.**

I. Introduction

One of the primary reasons for the large interest in neural networks is their capability to approximate a large class of continuous nonlinear maps from the collective action of very simple, autonomous processing units interconnected in simple ways. Neural networks have also attracted attention due to their inherently parallel and highly redundant processing architecture that makes it possible to develop parallel weight update laws. This parallelism makes it possible to effectively update a neural network on line. These properties make neural networks a viable paradigm for adaptive system identification and control of complex highly uncertain dynamical systems, and as a consequence the use of neural networks for identification and control has become an active area of research [1]–[9].

Manuscript received January 10, 2003; revised November 4, 2003. This work was supported in part by the National Science Foundation under Grant ECS-9496249 and in part by the Air Force Office of Scientific Research under Grants F49620-03-1-0178 and F49620-03-1-0443.

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Digital Object Identifier 10.1109/TNN.2004.841791

Modern complex engineering systems as well as biological and physiological systems are highly interconnected and mutually interdependent, both physically and through a multitude of information and communication networks. By properly formulating these systems in terms of subsystem interaction and energy/mass transfer, the dynamical models of many of these systems can be derived from mass, energy, and information balance considerations that involve dynamic states whose values are nonnegative. Hence, it follows from physical considerations that the state trajectory of such systems remains in the nonnegative orthant of the state–space for nonnegative initial conditions. Such systems are commonly referred to as *nonnegative dynamical systems* in [10]–[13]. A subclass of nonnegative dynamical systems are *compartmental systems* [12], [14]–[23]. Compartmental systems involve dynamical models that are characterized by conservation laws (e.g., mass and energy) capturing the exchange of material between coupled macroscopic subsystems known as compartments. Each compartment is assumed to be kinetically homogeneous; that is, any material entering the compartment is instantaneously mixed with the material of the compartment. The range of application of nonnegative systems and compartmental systems is quite large and includes biological, ecological, and chemical systems [16], [21], [24], [25]. Due to the severe complexities, nonlinearities, and uncertainties inherent in these systems, neural networks provide an ideal framework for online adaptive control because of their parallel processing flexibility and adaptability.

In this paper we develop a full-state feedback neural adaptive control framework for set-point regulation of nonlinear uncertain nonnegative and compartmental systems. Nonzero set-point regulation for nonnegative dynamical systems is a key design requirement since stabilization of nonnegative systems naturally deals with equilibrium points in the interior of the nonnegative orthant. The proposed framework is Lyapunov-based and guarantees ultimate boundedness of the error signals corresponding to the physical system states as well as the neural network weighting gains. The neuro adaptive controllers are constructed *without* requiring knowledge of the system dynamics while guaranteeing that the physical system states remain in the nonnegative orthant of the state–space. The proposed neuro control architecture is modular in the sense that if a nominal linear design model is available, the neuro adaptive controller can be augmented to the nominal design to account for system nonlinearities and system uncertainty. Furthermore, since in certain applications of nonnegative and compartmental systems (e.g., pharmacological systems for active drug administration) control (source) inputs as well as the system states need to be nonnegative, we also develop neuro adaptive controllers that guarantee the control signal as well as the physical system states remain nonnegative for nonnegative initial conditions. We note that neuro adaptive controllers for nonnegative dynamical systems have not been addressed in the literature. Our approach however, is related to the neuro adaptive control methods developed in [26]–[28]. Finally, the proposed neuro adaptive control framework is used to regulate the temperature of a continuously stirred tank reactor involving exothermic irreversible reactions.

The contents of the paper are as follows. In Section II, we provide mathematical preliminaries on nonnegative dynamical systems that are necessary for developing the main results of this paper. In Section III, we develop *new* Lyapunov-like theorems for partial boundedness and partial ultimate boundedness for nonlinear dynamical systems necessary for obtaining less conservative ultimate bounds for neuro adaptive controllers as compared to ultimate bounds derived using classical boundedness and ultimate boundedness notions. In Section IV, we present our main neuro adaptive control framework for adaptive set-point regulation of nonlinear uncertain nonnegative and compartmental systems. In Section V, we extend the results of Section IV to the case where control inputs are constrained to be nonnegative. To demonstrate the efficacy of the proposed neuro adaptive control framework, in Section VI, we apply our framework to control a continuously stirred tank reactor involving exothermic irreversible reactions. Finally, in Section VII, we draw some conclusions.

II. Mathematical Preliminaries

In this section, we introduce notation, several definitions, and some key results concerning linear and nonlinear nonnegative dynamical systems [12], [22], [29], [30] that are necessary for developing the main results of this paper. Specifically, for $x \in \mathbb{R}^n$ we write $x \geq\geq 0$ (resp., $x \gg 0$) to indicate that every component of x is nonnegative (resp., positive). In this case, we say that x is *nonnegative* or *positive*, respectively. Likewise, $A \in \mathbb{R}^{n\times m}$ is *nonnegative*[1] or *positive* if every entry of A is nonnegative or positive, respectively, which is written as $A \geq\geq 0$ or $A \gg 0$, respectively. Let $\overline{\mathbb{R}}_+^n$ and $\mathbb{R}_+^n$ denote the nonnegative and positive orthants of $\mathbb{R}^n$; that is, if $x \in \mathbb{R}^n$, then $x \in \overline{\mathbb{R}}_+^n$ and $x \in \mathbb{R}_+^n$ are equivalent, respectively, to $x \geq\geq 0$ and $x \gg 0$. Finally, we write $(\ )^{\mathrm{T}}$ to denote transpose, $\mathrm{tr}(\cdot)$ for the trace operator, $\lambda_{\min}(\cdot)$ to denote the minimum eigenvalue of a Hermitian matrix, $\|\cdot\|$ for a vector norm, $\|\cdot\|_{\mathrm{F}}$ for the Frobenius matrix norm, and $V'(x)$ for the Fréchet derivative of V at x. The following definition introduces the notion of a nonnegative (resp., positive) function.

Definition 2.1: Let $T > 0$. A real function $u : [0, T] \to \mathbb{R}^m$ is a *nonnegative* (resp., *positive*) *function* if $u(t) \geq\geq 0$ (resp., $u(t) \gg 0$) on the interval $[0, T]$.

The next definition introduces the notions of essentially nonnegative matrices and compartmental matrices.

[1]In this paper it is important to distinguish between a square nonnegative (resp., positive) matrix and a nonnegative–definite (resp., positive–definite) matrix.

Definition 2.2 ([12], [22]): Let $A \in \mathbb{R}^{n\times n}$. A is *essentially nonnegative* if $A_{(i,j)} \geq 0$, $i, j = 1, \ldots, n$, $i \neq j$. A is *compartmental* if A is essentially nonnegative and $\sum_{i=1}^n A_{(i,j)} \leq 0$, $j = 1, \ldots, n$.

Next, consider the controlled linear dynamical system

$$\dot{x}(t) = Ax(t) + Bu(t), \quad x(0) = x_0, \quad t \geq 0 \tag{1}$$

where

$$B = \begin{bmatrix} \hat{B} \\ 0_{(n-m)\times m} \end{bmatrix} \tag{2}$$

$A \in \mathbb{R}^{n\times n}$ is essentially nonnegative and $\hat{B} \in \mathbb{R}^{m\times m}$ is nonnegative such that $\mathrm{rank}\,\hat{B} = m$. The following theorem shows that linear stabilizable nonnegative systems possess asymptotically stable zero dynamics with $\hat{x} \triangleq [x_1, \ldots, x_m]$ viewed as the output. For the statement of this result let $\mathrm{spec}(A)$ denote the spectrum of A, let $\overline{\mathbb{C}}_+ \triangleq \{s \in \mathbb{C} : \mathrm{Re}[s] \geq 0\}$, and let $A \in \mathbb{R}^{n\times n}$ in (1) be partitioned as

$$A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \tag{3}$$

where $A_{11} \in \mathbb{R}^{m\times m}$ is essentially nonnegative, $A_{12} \in \mathbb{R}^{m\times(n-m)}$ is nonnegative, $A_{21} \in \mathbb{R}^{(n-m)\times m}$ is nonnegative, and $A_{22} \in \mathbb{R}^{(n-m)\times(n-m)}$ is essentially nonnegative.

Theorem 2.1: Consider the linear dynamical system $\mathcal{G}$ given by (1) where $A \in \mathbb{R}^{n\times n}$ is essentially nonnegative and partitioned as in (3), and $B \in \mathbb{R}^{n\times m}$ is nonnegative and is partitioned as in (2) with $\mathrm{rank}\,\hat{B} = m$. Then there exists a gain matrix $K \in \mathbb{R}^{m\times n}$ such that $A + BK$ is essentially nonnegative and asymptotically stable if and only if A_{22} is asymptotically stable.

Proof: First, let K be partitioned as $K = [K_1, K_2]$, where $K_1 \in \mathbb{R}^{m\times m}$ and $K_2 \in \mathbb{R}^{m\times(n-m)}$, and note that

$$(A + BK)^{\mathrm{T}} = \begin{bmatrix} (A_{11} + \hat{B}K_1)^{\mathrm{T}} & A_{21}^{\mathrm{T}} \\ (A_{12} + \hat{B}K_2)^{\mathrm{T}} & A_{22}^{\mathrm{T}} \end{bmatrix}.$$

Assume that $A + BK$ is essentially nonnegative and asymptotically stable and suppose, *ad absurdum*, A_{22} is not asymptotically stable. Then, it follows from [12, Th. 3.1] that there does not exist a positive vector $p_2 \in \mathbb{R}_+^{n-m}$ such that $A_{22}^{\mathrm{T}} p_2 \ll 0$. Next, since $A_{12} + \hat{B}K_2$ is nonnegative it follows that $(A_{12} + \hat{B}K_2)^{\mathrm{T}} p_1 \geq\geq 0$ for any positive vector $p_1 \in \mathbb{R}_+^m$. Thus, there does not exist a positive vector $p \triangleq [p_1^{\mathrm{T}}, p_2^{\mathrm{T}}]^{\mathrm{T}}$ such that $(A + BK)^{\mathrm{T}} p \ll 0$ and, hence, it follows from [12, Th. 3.1] that $A + BK$ is not asymptotically stable leading to a contradiction. Hence, A_{22} is asymptotically stable. Conversely, suppose A_{22} is asymptotically stable. Then taking $K_1 = \hat{B}^{-1}(A_{\mathrm{s}} - A_{11})$ and $K_2 = -\hat{B}^{-1} A_{12}$, where A_{s} is essentially nonnegative and asymptotically stable, it follows that $\mathrm{spec}(A + BK) \cap \overline{\mathbb{C}}_+ = [\mathrm{spec}(A_{\mathrm{s}}) \cup \mathrm{spec}(A_{22})] \cap \overline{\mathbb{C}}_+ = \varnothing$ and, hence, $A + BK$ is essentially nonnegative and asymptotically stable. □

The following definition introduces the notion of essentially nonnegative vector fields [12], [31].

Definition 2.3: Let $f = [f_1, \ldots, f_n]^{\mathrm{T}} : \mathcal{D} \to \mathbb{R}^n$, where $\mathcal{D}$ is an open subset of $\mathbb{R}^n$ that contains $\overline{\mathbb{R}}_+^n$. Then f is *essentially nonnegative with respect to* $\hat{x} \triangleq [x_1, \ldots, x_m]^{\mathrm{T}}$, $m \leq n$, if $f_i(x) \geq 0$ for all $i = 1, \ldots, m$, and $x \in \overline{\mathbb{R}}_+^n$ such that $x_i =$

0, $i = 1, \ldots, m$, where x_i denotes the ith element of x. f is *essentially nonnegative* if $f_i(x) \geq 0$ for all $i = 1, \ldots, n$, and $x \in \overline{\mathbb{R}}_+^n$ such that $x_i = 0$.

In this paper we consider controlled time-varying nonlinear dynamical systems of the form

$$\dot{x}(t) = f(t, x(t)) + G(x(t))u(t), \quad x(t_0) = x_0, \quad t \geq t_0 \quad (4)$$

where $x(t) \in \mathbb{R}^n$, $t \geq 0$, $u(t) \in \mathbb{R}^m$, $t \geq 0$, $f : [t_0, \infty) \times \mathbb{R}^n \to \mathbb{R}^n$ is continuous in t and Lipschitz continuous in x on $[t_0, \infty) \times \mathbb{R}^n$ and satisfies $f(t, 0) = 0$, $t \geq t_0$, and $G : \mathbb{R}^n \to \mathbb{R}^{n \times m}$.

The following definition and proposition are needed for the main results of the paper.

Definition 2.4: The nonlinear dynamical system given by (4) is *nonnegative* if for every $x(0) \in \overline{\mathbb{R}}_+^n$ and $u(t) \geq\geq 0$, $t \geq 0$, the solution $x(t)$, $t \geq 0$, to (4) is nonnegative.

Proposition 2.1: Consider the time-varying dynamical system (4) where $f(t, \cdot) : \mathbb{R}^n \to \mathbb{R}^n$ is Lipschitz continuous on $\mathbb{R}^n$ for all $t \in [t_0, \infty)$ and $f(\cdot, x) : [t_0, \infty) \to \mathbb{R}^n$ is continuous on $[t_0, \infty)$ for all $x \in \mathbb{R}^n$. If for every $t \in [t_0, \infty)$, $f : [t_0, \infty) \times \mathbb{R}^n \to \mathbb{R}^n$ is essentially nonnegative and $G : \mathbb{R}^n \to \mathbb{R}^{n \times m}$ is nonnegative, then the solution $x(t)$, $t \geq t_0$, to (4) is nonnegative.

Proof: The result is a direct consequence of [12, Prop 7.1] by equivalently representing the time-varying system (4) as an autonomous nonlinear system by appending another state to represent time. Specifically, defining $y(t - t_0) \triangleq x(t)$ and $y_{n+1}(t - t_0) \triangleq t$, it follows that the solution $x(t)$, $t \geq t_0$, to (4) can be equivalently characterized by the solution $y(\tau)$, $\tau \geq 0$, where $\tau \triangleq t - t_0$, to the nonlinear autonomous system

$$\dot{y}(\tau) = f(y_{n+1}(\tau), y(\tau)) + G(y(\tau))\hat{u}(\tau), \quad y(0) = y_0, \quad \tau \geq 0 \quad (5)$$

$$\dot{y}_{n+1}(\tau) = 1, \quad y_{n+1}(0) = t_0 \quad (6)$$

where $\dot{y}(\cdot)$ and $\dot{y}_{n+1}(\cdot)$ denote differentiation with respect to τ and $\hat{u}(\tau) \triangleq u(\tau + t_0)$. Now, since $\dot{y}_i(\tau) \geq 0$, $\tau \geq 0$, whenever $y_i(\tau) = 0$ for $i = 1, \ldots, n+1$, and $G(y(\tau))\hat{u}(\tau) \geq\geq 0$, $\tau \geq 0$, the result is a direct consequence of [12, Prop 7.1]. □

It follows from Proposition 2.1 that a nonnegative input signal $G(x(t))u(t)$, $t \geq 0$, is sufficient to guarantee the nonnegativity of the state of (4).

III. Partial Boundedness and Partial Ultimate Boundedness

In this section, we present Lyapunov-like theorems for *partial boundedness* and *partial ultimate boundedness* of nonlinear dynamical systems. These notions allow us to develop less conservative ultimate bounds for neuro adaptive controllers as compared to ultimate bounds derived using classical boundedness and ultimate boundedness notions. Specifically, consider the nonlinear autonomous interconnected dynamical system

$$\dot{x}_1(t) = f_1(x_1(t), x_2(t)), \quad x_1(0) = x_{10}, \quad t \in \mathcal{I}_{x_{10}, x_{20}} \quad (7)$$

$$\dot{x}_2(t) = f_2(x_1(t), x_2(t)), \quad x_2(0) = x_{20} \quad (8)$$

where $x_1 \in \mathcal{D}$, $\mathcal{D} \subseteq \mathbb{R}^{n_1}$ is an open set such that $0 \in \mathcal{D}$, $x_2 \in \mathbb{R}^{n_2}$, $f_1 : \mathcal{D} \times \mathbb{R}^{n_2} \to \mathbb{R}^{n_1}$ is such that, for every $x_2 \in \mathbb{R}^{n_2}$, $f_1(0, x_2) = 0$ and $f_1(\cdot, x_2)$ is locally Lipschitz in x_1, $f_2 : \mathcal{D} \times \mathbb{R}^{n_2} \to \mathbb{R}^{n_2}$ is such that, for every $x_1 \in \mathcal{D}$, $f_2(x_1, \cdot)$ is locally Lipschitz in x_2, and $\mathcal{I}_{x_{10}, x_{20}} \triangleq [0, \tau_{x_{10}, x_{20}})$, $0 < \tau_{x_{10}, x_{20}} \leq \infty$, is the maximal interval of existence for the solution $(x_1(t), x_2(t))$, $t \in \mathcal{I}_{x_{10}, x_{20}}$, to (7), (8). Note that under the previous assumptions the solution $(x_1(t), x_2(t))$ to (7), (8) exists and is unique over $\mathcal{I}_{x_{10}, x_{20}}$. For the following definition we assume that $\mathcal{I}_{x_{10}, x_{20}} = [0, \infty)$.

Definition 3.1:

i) The nonlinear dynamical system (7), (8) is *bounded with respect to x_1 uniformly in x_{20}* if there exists $\gamma > 0$ such that, for every $\delta \in (0, \gamma)$, there exists $\varepsilon = \varepsilon(\delta) > 0$ such that $\|x_{10}\| < \delta$ implies $\|x_1(t)\| < \varepsilon$, $t \geq 0$. The nonlinear dynamical system (7), (8) is *globally bounded with respect to x_1 uniformly in x_{20}* if, for every $\delta \in (0, \infty)$, there exists $\varepsilon = \varepsilon(\delta) > 0$ such that $\|x_{10}\| < \delta$ implies $\|x_1(t)\| < \varepsilon$, $t \geq 0$.

ii) The nonlinear dynamical system (7), (8) is *ultimately bounded with respect to x_1 uniformly in x_{20} with ultimate bound ε* if there exists $\gamma > 0$ such that, for every $\delta \in (0, \gamma)$, there exists $T = T(\delta, \varepsilon) > 0$ such that $\|x_{10}\| < \delta$ implies $\|x_1(t)\| < \varepsilon$, $t \geq T$. The nonlinear dynamical system (7), (8) is *globally ultimately bounded with respect to x_1 uniformly in x_{20} with ultimate bound ε* if, for every $\delta \in (0, \infty)$, there exists $T = T(\delta, \varepsilon) > 0$ such that $\|x_{10}\| < \delta$ implies $\|x_1(t)\| < \varepsilon$, $t \geq T$.

Note that if a nonlinear dynamical system is (globally) bounded with respect to x_1 uniformly in x_{20}, then there exists $\varepsilon > 0$ such that it is (globally) ultimately bounded with respect to x_1 uniformly in x_{20} with an ultimate bound ε. Conversely, if a nonlinear dynamical system is (globally) ultimately bounded with respect to x_1 uniformly in x_{20} with an ultimate bound ε, then it is (globally) bounded with respect to x_1 uniformly in x_{20}. The following results present Lyapunov-like theorems for partial boundedness and partial ultimate boundedness. For these results define $\dot{V}(x_1, x_2) \triangleq V'(x_1, x_2)f(x_1, x_2)$, where $f(x_1, x_2) \triangleq [f_1^{\mathrm{T}}(x_1, x_2), f_2^{\mathrm{T}}(x_1, x_2)]^{\mathrm{T}}$ and $V : \mathcal{D} \times \mathbb{R}^{n_2} \to \mathbb{R}$ is a given continuously differentiable function. Furthermore, let $\mathcal{B}_\delta(x)$, $x \in \mathbb{R}^n$, $\delta > 0$, denote the open ball centered at x with radius δ and let $\overline{\mathcal{B}}_\delta(x)$ denote the closure of $\mathcal{B}_\delta(x)$.

Theorem 3.1: Consider the nonlinear dynamical system (7), (8). Assume there exist a continuously differentiable function $V : \mathcal{D} \times \mathbb{R}^{n_2} \to \mathbb{R}$ and class $\mathcal{K}$ functions $\alpha(\cdot)$, $\beta(\cdot)$ such that

$$\alpha(\|x_1\|) \leq V(x_1, x_2) \leq \beta(\|x_1\|), x_1 \in \mathcal{D}, x_2 \in \mathbb{R}^{n_2} \quad (9)$$

$$\dot{V}(x_1, x_2) \leq 0, \quad x_1 \in \mathcal{D}, \quad \|x_1\| \geq \mu, \quad x_2 \in \mathbb{R}^{n_2} \quad (10)$$

where $\mu > 0$ is such that $\mathcal{B}_{\alpha^{-1}(\eta)}(0) \subset \mathcal{D}$ with $\eta \geq \beta(\mu)$. Then the nonlinear dynamical system (7), (8) is bounded with respect to x_1 uniformly in x_{20}. Furthermore, for every $\delta \in (0, \gamma)$, $x_{10} \in \overline{\mathcal{B}}_\delta(0)$ implies that $\|x_1(t)\| \leq \varepsilon$, where

$$\varepsilon(\delta) \triangleq \begin{cases} \alpha^{-1}(\beta(\delta)), & \delta \in (\mu, \gamma) \\ \alpha^{-1}(\eta), & \delta \in (0, \mu] \end{cases} \quad (11)$$

and $\gamma \triangleq \sup\{r > 0 : \mathcal{B}_{\alpha^{-1}(\beta(r))}(0) \subset \mathcal{D}\}$. If, in addition, $\mathcal{D} = \mathbb{R}^{n_1}$ and $\alpha(\cdot)$ is a class $\mathcal{K}_\infty$ function, then the nonlinear dynamical system (7), (8) is globally bounded with respect to x_1 uniformly in x_{20} and for every $x_{10} \in \mathbb{R}^{n_1}$, $\|x_1(t)\| \leq \varepsilon$, $t \geq 0$, where ε is given by (11) with $\delta = \|x_{10}\|$.

Proof: First, let $\delta \in (0,\mu]$ and assume $\|x_{10}\| \le \delta$. If $\|x_1(t)\| \le \mu, t \ge 0$, then it follows from (9) that $\|x_1(t)\| \le \mu \le \alpha^{-1}(\beta(\mu)) \le \alpha^{-1}(\eta), t \ge 0$. Alternatively, if there exists $T > 0$ such that $\|x_1(T)\| > \mu$, then it follows from the continuity of $x_1(\cdot)$ that there exists $\tau < T$ such that $\|x_1(\tau)\| = \mu$ and $\|x_1(t)\| \ge \mu$, $t \in [\tau, T]$. Hence, it follows from (9) and (10) that:

$$\alpha(\|x_1(T)\|) \le V(x_1(T), x_2(T)) \le V(x_1(\tau), x_2(\tau)) \le \beta(\mu) \le \eta$$

which implies that $\|x_1(T)\| \le \alpha^{-1}(\eta)$. Next, let $\delta \in (\mu, \gamma)$ and assume $x_{10} \in \overline{\mathcal{B}}_\delta(0)$ and $\|x_{10}\| > \mu$. Now, for every $\hat{t} > 0$ such that $\|x_1(t)\| \ge \mu, t \in [0, \hat{t}]$, it follows from (9) and (10) that

$$\alpha(\|x_1(t)\|) \le V(x_1(t), x_2(t)) \le V(x_{10}, x_{20}) \le \beta(\delta), \quad t \ge 0$$

which implies that $\|x_1(t)\| \le \alpha^{-1}(\beta(\delta))$, $t \in [0, \hat{t}]$. Next, if there exists $T > 0$ such that $\|x_1(T)\| \le \mu$, then it follows as in the proof of the first case given previously that $\|x_1(t)\| \le \alpha^{-1}(\eta), t \ge T$. Hence, if $x_{10} \in \mathcal{B}_\delta(0) \backslash \mathcal{B}_\mu(0)$, then $\|x_1(t)\| \le \alpha^{-1}(\beta(\delta))$, $t \ge 0$. Finally, if $\mathcal{D} = \mathbb{R}^{n_1}$ and $\alpha(\cdot)$ is a class $\mathcal{K}_\infty$ function it follows that $\beta(\cdot)$ is a class $\mathcal{K}_\infty$ function and, hence, $\gamma = \infty$. Hence, the nonlinear dynamical system (7), (8) is globally bounded with respect to x_1 uniformly in x_{20}. □

Theorem 3.2: Consider the nonlinear dynamical system (7), (8). Assume there exist a continuously differentiable function $V : \mathcal{D} \times \mathbb{R}^{n_2} \to \mathbb{R}$ and class $\mathcal{K}$ functions $\alpha(\cdot)$, $\beta(\cdot)$ such that (9) holds. Furthermore, assume that there exists a continuous, positive–definite function $W : \mathcal{D} \to \mathbb{R}$ such that $W(x_1) > 0$, $\|x_1\| > \mu$, and

$$\dot{V}(x_1, x_2) \le -W(x_1), \quad x_1 \in \mathcal{D}, \quad \|x_1\| > \mu, \quad x_2 \in \mathbb{R}^{n_2} \tag{12}$$

where $\mu > 0$ is such that $\mathcal{B}_{\alpha^{-1}(\eta)}(0) \subset \mathcal{D}$ with $\eta > \beta(\mu)$. Then the nonlinear dynamical system (7), (8) is ultimately bounded with respect to x_1 uniformly in x_{20} with ultimate bound $\varepsilon \triangleq \alpha^{-1}(\eta)$. Furthermore, $\limsup_{t\to\infty} \|x_1(t)\| \le \alpha^{-1}(\beta(\mu))$. If, in addition, $\mathcal{D} = \mathbb{R}^n$ and $\alpha(\cdot)$ is a class $\mathcal{K}_\infty$ function, then the nonlinear dynamical system (7), (8) is globally ultimately bounded with respect to x_1 uniformly in x_{20} with ultimate bound ε.

Proof: First, let $\delta \in (0,\mu]$ and assume $\|x_{10}\| \le \delta$. As in the proof of Theorem 3.1, it follows that $\|x_1(t)\| \le \alpha^{-1}(\eta) = \varepsilon$, $t \ge 0$. Next, let $\delta \in (\mu, \gamma)$, where $\gamma \triangleq \sup\{r > 0 : \mathcal{B}_{\alpha^{-1}(\beta(r))}(0) \subset \mathcal{D}\}$ and assume $x_{10} \in \mathcal{B}_\delta(0)$ and $\|x_{10}\| > \mu$. In this case, it follows from Theorem 3.1 that $\|x_1(t)\| \le \alpha^{-1}(\beta(\delta))$, $t \ge 0$. Suppose, *ad absurdum*, $\|x_1(t)\| \ge \beta^{-1}(\eta)$, $t \ge 0$, or, equivalently, $x_1(t) \in \mathcal{O} \triangleq \mathcal{B}_{\alpha^{-1}(\beta(\delta))}(0) \backslash \mathcal{B}_{\beta^{-1}(\eta)}(0)$, $t \ge 0$. Since $\overline{\mathcal{O}}$ is compact and $W(\cdot)$ is continuous and $W(x_1) > 0$, $\|x_1\| \ge \beta^{-1}(\eta) > \mu$, it follows from Weierstrass' theorem [32, p. 154] that $k \triangleq \min_{x_1 \in \overline{\mathcal{O}}} W(x_1) > 0$ exists. Hence, it follows from (12) that:

$$V(x_1(t), x_2(t)) \le V(x_{10}, x_{20}) - kt, \quad t \ge 0 \tag{13}$$

which implies that

$$\alpha(\|x_1(t)\|) \le \beta(\|x_{10}\|) - kt \le \beta(\delta) - kt, \quad t \ge 0. \tag{14}$$

Now, letting $t > \beta(\delta)/k$ it follows that $\alpha(\|x_1(t)\|) < 0$ which is a contradiction. Hence, there exists $T = T(\delta, \eta) > 0$ such that $\|x_1(T)\| < \beta^{-1}(\eta)$. Thus, it follows from Theorem 3.1 that $\|x_1(t)\| \le \alpha^{-1}(\beta(\beta^{-1}(\eta))) = \alpha^{-1}(\eta)$, $t \ge T$, which proves that the nonlinear dynamical system (7), (8) is ultimately bounded with respect to x_1 uniformly in x_{20} with ultimate bound $\varepsilon = \alpha^{-1}(\eta)$. Furthermore, $\limsup_{t\to\infty} \|x_1(t)\| \le \alpha^{-1}(\beta(\mu))$. Finally, if $\mathcal{D} = \mathbb{R}^{n_1}$ and $\alpha(\cdot)$ is a class $\mathcal{K}_\infty$ function it follows that $\beta(\cdot)$ is a class $\mathcal{K}_\infty$ function and, hence, $\gamma = \infty$. Hence, the nonlinear dynamical system (7), (8) is globally ultimately bounded with respect to x_1 uniformly in x_{20} with ultimate bound ε. □

The following result on ultimate boundedness of interconnected systems is needed for the main theorems in this paper.

Proposition 3.1: Consider the nonlinear interconnected dynamical system (7), (8). If (8) is input-to-state stable with x_1 viewed as the input and (7), (8) is ultimately bounded with respect to x_1 uniformly in x_{20}, then the solution $(x_1(t), x_2(t))$, $t \ge 0$, of the interconnected dynamical system (7), (8) is ultimately bounded.

Proof: Since (7), (8) is ultimately bounded with respect to x_1 (uniformly in x_{20}), there exist positive constants ε and $T = T(\delta, \varepsilon)$ such that $\|x_1(t)\| < \varepsilon$, $t \ge T$. Furthermore, since (8) is input-to-state stable with x_1 viewed as the input, it follows that $x_2(T)$ is finite and, hence, there exist a class $\mathcal{KL}$ function $\eta(\cdot,\cdot)$ and a class $\mathcal{K}$ function $\gamma(\cdot)$ such that

$$\begin{aligned}\|x_2(t)\| &\le \eta(\|x_2(T)\|, t-T) + \gamma\Big(\sup_{T \le \tau \le t} \|x_1(\tau)\|\Big) \\ &= \eta(\|x_2(T)\|, t-T) + \gamma(\varepsilon) \\ &\le \eta(\|x_2(T)\|, 0) + \gamma(\varepsilon), \quad t \ge T\end{aligned} \tag{15}$$

which proves that the solution $(x_1(t), x_2(t))$, $t \ge 0$, to (7), (8) is ultimately bounded. □

IV. Neural Adaptive Control for Nonlinear Nonnegative Uncertain Systems

In this section, we consider the problem of characterizing neural adaptive feedback control laws for nonlinear nonnegative and compartmental uncertain dynamical systems to achieve *set-point* regulation in the nonnegative orthant. Specifically, consider the controlled nonlinear uncertain dynamical system $\mathcal{G}$ given by

$$\dot{x}(t) = f_x(x(t), z(t)) + G(x(t), z(t))u(t), \quad x(0) = x_0, \quad t \ge 0 \tag{16}$$

$$\dot{z}(t) = f_z(x(t), z(t)), \quad z(0) = z_0 \tag{17}$$

where $x(t) \in \mathbb{R}^{n_x}$, $t \ge 0$, and $z(t) \in \mathbb{R}^{n_z}$, $t \ge 0$, are the state vectors, $u(t) \in \mathbb{R}^m$, $t \ge 0$, is the control input, $f_x : \mathbb{R}^{n_x} \times \mathbb{R}^{n_z} \to \mathbb{R}^{n_x}$ is essentially nonnegative with respect to x but otherwise unknown and satisfies $f_x(0, z) = 0$, $z \in \mathbb{R}^{n_z}$, $f_z : \mathbb{R}^{n_x} \times \mathbb{R}^{n_z} \to \mathbb{R}^{n_z}$ is essentially nonnegative with respect to z but otherwise unknown and satisfies $f_z(x, 0) = 0$, $x \in \mathbb{R}^{n_x}$, and $G : \mathbb{R}^{n_x} \times \mathbb{R}^{n_z} \to \mathbb{R}^{n_x \times m}$ is a known nonnegative input matrix function. Here, we assume that we have m control inputs so that the input matrix function is given by

$$G(x, z) = \begin{bmatrix} B_{\mathrm{u}} G_n(x, z) \\ 0_{(n_x - m)\times m} \end{bmatrix} \tag{18}$$

where $B_{\rm u} = \mathrm{diag}[b_1, \ldots, b_m]$ is a positive diagonal matrix and $G_n : \mathbb{R}^{n_x} \times \mathbb{R}^{n_z} \to \mathbb{R}^{m\times m}$ is a nonnegative matrix function such that $\det G_n(x,z) \neq 0$, $(x,z) \in \mathbb{R}^{n_x} \times \mathbb{R}^{n_z}$. The control input $u(\cdot)$ in (16) is restricted to the class of *admissible controls* consisting of measurable functions such that $u(t) \in \mathbb{R}^m$, $t \geq 0$. In this section, we do not place any restriction on the sign of the control signal and design a neuro adaptive controller that guarantees that the system states remain in the nonnegative orthant of the state–space for nonnegative initial conditions and are ultimately bounded in the neighborhood of a desired equilibrium point.

In this paper, we assume that $f_x(\cdot,\cdot)$ and $f_z(\cdot,\cdot)$ are unknown functions with $f_x(\cdot,\cdot)$ given by

$$f_x(x,z) = Ax + \Delta f(x,z) \tag{19}$$

where $A \in \mathbb{R}^{n_x \times n_x}$ is a known essentially nonnegative matrix and $\Delta f : \mathbb{R}^{n_x} \times \mathbb{R}^{n_z} \to \mathbb{R}^{n_x}$ is an unknown essentially nonnegative function with respect to x and belongs to the uncertainty set $\mathcal{F}$ given by

$$\begin{aligned}\mathcal{F} = \{&\Delta f : \mathbb{R}^{n_x} \times \mathbb{R}^{n_z} \to \mathbb{R}^{n_x} :\\ &\Delta f(x,z) = B\delta(x,z),\ (x,z) \in \mathbb{R}^{n_x} \times \mathbb{R}^{n_z}\}\end{aligned} \tag{20}$$

where $B \triangleq [B_{\rm u}, 0_{m\times(n-m)}]^{\rm T}$ and $\delta : \mathbb{R}^{n_x} \times \mathbb{R}^{n_z} \to \mathbb{R}^m$ is an uncertain continuous function such that $\delta(x,z)$ is essentially nonnegative with respect to x and $\delta'(x,z)$ is bounded for all $(x,z) \in \mathcal{D}_{{\rm c}x} \times \mathcal{D}_{{\rm c}z}$. Furthermore, we assume that for a given $x_{\rm e} \in \mathbb{R}_+^{n_x}$ there exist $z_{\rm e} \in \overline{\mathbb{R}}_+^{n_z}$ and $u_{\rm e} \in \overline{\mathbb{R}}_+^{m}$ such that

$$0 = Ax_{\rm e} + \Delta f(x_{\rm e}, z_{\rm e}) + G(x_{\rm e}, z_{\rm e})u_{\rm e} \tag{21}$$
$$0 = f_z(x_{\rm e}, z_{\rm e}). \tag{22}$$

In addition, we assume that (17) is input-to-state stable at $z(t) \equiv z_{\rm e}$ with $x(t) - x_{\rm e}$ viewed as the input; that is, there exist a class $\mathcal{KL}$ function $\eta(\cdot,\cdot)$ and a class $\mathcal{K}$ function $\gamma(\cdot)$ such that

$$\|z(t) - z_{\rm e}\| \leq \eta(\|z_0 - z_{\rm e}\|, t) + \gamma\left(\sup_{0\leq\tau\leq t} \|x(\tau) - x_{\rm e}\|\right), t \geq 0 \tag{23}$$

where $\|\cdot\|$ denotes the Euclidean vector norm. Unless otherwise stated, henceforth, we use $\|\cdot\|$ to denote the Euclidean vector norm. Note that $(x_{\rm e}, z_{\rm e}) \in \mathbb{R}_+^{n_x} \times \overline{\mathbb{R}}_+^{n_z}$ is an equilibrium point of (16), (17) if and only if there exists $u_{\rm e} \in \overline{\mathbb{R}}_+^{m}$ such that (21), (22) hold. Furthermore, we assume that for a given $\varepsilon_i^* > 0$, the ith component of the vector function $\delta(x,z) - \delta(x_{\rm e}, z_{\rm e}) - G_n(x_{\rm e}, z_{\rm e})u_{\rm e}$ can be approximated over a compact set $\mathcal{D}_{{\rm c}x} \times \mathcal{D}_{{\rm c}z} \subset \overline{\mathbb{R}}_+^{n_x} \times \overline{\mathbb{R}}_+^{n_z}$ by a linear in the parameters neural network up to a desired accuracy so that for $i = 1, \ldots, m$, there exists $\varepsilon_i(\cdot,\cdot)$ such that $|\varepsilon_i(x,z)| < \varepsilon_i^*$, $(x,z) \in \mathcal{D}_{{\rm c}x} \times \mathcal{D}_{{\rm c}z}$, and

$$\begin{aligned}\delta_i(x,z) - \delta_i(x_{\rm e}, z_{\rm e}) - [G_n(x_{\rm e}, z_{\rm e})u_{\rm e}]_i &= W_i^{\rm T}\sigma_i(x,z) + \varepsilon_i(x,z)\\ &(x,z) \in \mathcal{D}_{{\rm c}x} \times \mathcal{D}_{{\rm c}z}\end{aligned} \tag{24}$$

where $W_i \in \mathbb{R}^{s_i}$, $i = 1, \ldots, m$, are optimal *unknown* (constant) weights that minimize the approximation error over $\mathcal{D}_{{\rm c}x} \times \mathcal{D}_{{\rm c}z}$, $\sigma_i : \mathbb{R}^{n_x} \times \mathbb{R}^{n_z} \to \mathbb{R}^{s_i}$, $i = 1, \ldots, m$, are a set of basis functions such that each component of $\sigma_i(\cdot,\cdot)$ takes values between 0 and 1 and $\sigma'(x,z)$, $(x,z) \in \mathcal{D}_{{\rm c}x} \times \mathcal{D}_{{\rm c}z}$, is bounded, $\varepsilon_i : \mathcal{D}_{{\rm c}x} \times \mathcal{D}_{{\rm c}z} \to \mathbb{R}$, $i = 1, \ldots, m$, are the modeling errors, and $\|W_i\| \leq w_i^*$, where w_i^*, $i = 1, \ldots, m$, are bounds for the optimal weights W_i, $i = 1, \ldots, m$. Since $f_x(\cdot,\cdot)$ is continuous, we can choose $\sigma_i(\cdot,\cdot)$, $i = 1, \ldots, m$, from a linear space $\mathcal{X}$ of continuous functions that forms an algebra and separates points in $\mathcal{D}_{{\rm c}x} \times \mathcal{D}_{{\rm c}z}$. In this case, it follows from the Stone-Weierstrass theorem [32, p. 212] that $\mathcal{X}$ is a dense subset of the set of continuous functions on $\mathcal{D}_{{\rm c}x} \times \mathcal{D}_{{\rm c}z}$. Now, as is the case in the standard neuro adaptive control literature [6], we can construct the signal $u_{{\rm ad}i} = \hat{W}_i^{\rm T}\sigma_i(x,z)$ involving the estimates of the optimal weights as our adaptive control signal. However, even though $\hat{W}_i^{\rm T}\sigma_i(x,z)$, $i = 1, \ldots, m$, provide adaptive cancellation of the system uncertainty, it does not necessarily guarantee that the state trajectory of the closed-loop system remains in the nonnegative orthant of the state space for nonnegative initial conditions. To ensure nonnegativity of the closed-loop plant states, the adaptive control signal is assumed to be of the form $\hat{W}_i^{\rm T}\hat{\sigma}_i(x,z)$, $i = 1, \ldots, m$, where $\hat{\sigma}_i : \mathbb{R}^{n_x} \times \mathbb{R}^{n_z} \to \mathbb{R}^{s_i}$ is such that each component of $\hat{\sigma}_i(\cdot,\cdot)$ takes values between 0 and 1, $\hat{\sigma}_i'(x,z)$, $(x,z) \in \mathcal{D}_{{\rm c}x} \times \mathcal{D}_{{\rm c}z}$, is bounded, and $\hat{\sigma}_i(x,z) = 0$ whenever $x_i = 0$ for all $i = 1, \ldots, m$. This set of functions do not generate an algebra in $\mathcal{X}$ and, hence, if used as an approximator for $\delta_i(\cdot,\cdot)$, $i = 1, \ldots, m$, will generate additional conservatism in the ultimate bound guarantees provided by the neural network controller. In particular, since each component of $\sigma_i(\cdot,\cdot)$ and $\hat{\sigma}_i(\cdot,\cdot)$ takes values between 0 and 1, it follows that:

$$\|\sigma_i(x,z) - \hat{\sigma}_i(x,z)\| \leq \sqrt{s_i}, \quad (x,z) \in \mathcal{D}_{{\rm c}x} \times \mathcal{D}_{{\rm c}z} \quad i = 1, \ldots, m. \tag{25}$$

This upper bound will be used in the analysis of Theorem 4.1 in the following.

For the remainder of the paper we assume that there exists a gain matrix $K \in \mathbb{R}^{m\times n_x}$ such that $A + BK$ is essentially nonnegative and asymptotically stable, where A and B have the forms of (3) and (2), respectively. Now, partitioning the state in (16) as $x = [x_1^{\rm T}, x_2^{\rm T}]^{\rm T}$, where $x_1 \in \mathbb{R}^m$ and $x_2 \in \mathbb{R}^{n_x - m}$, and using (18), it follows that (16) and (17) can be written as

$$\begin{aligned}\dot{x}_1(t) = &A_{11}x_1(t) + A_{12}x_2(t) + \Delta f(x_1(t), x_2(t), z(t))\\ &+ B_{\rm u}G_n(x_1(t), x_2(t), z(t))u(t), \quad x_1(0) = x_{10}\\ &t \geq 0\end{aligned} \tag{26}$$
$$\dot{x}_2(t) = A_{21}x_1(t) + A_{22}x_2(t), \quad x_2(0) = x_{20} \tag{27}$$
$$\dot{z}(t) = f_z(x_1(t), x_2(t), z(t)), z(0) = z_0. \tag{28}$$

Thus, since $A + BK$ is essentially nonnegative and asymptotically stable, it follows from Theorem 2.1 that the solution $x_2(t) \equiv x_{2{\rm e}} \in \mathbb{R}_+^{n_x - m}$ of (27) with $x_1(t) \equiv x_{1{\rm e}} \in \mathbb{R}_+^m$, where $x_{1{\rm e}}$ and $x_{2{\rm e}}$ satisfy $0 = A_{21}x_{1{\rm e}} + A_{22}x_{2{\rm e}}$, is globally exponentially stable and, hence, (27) is input-to-state stable at

$x_2(t) \equiv x_{2\mathrm{e}}$ with $x_1(t) - x_{1\mathrm{e}}$ viewed as the input. Thus, in this paper we assume that the dynamics (27) can be included in (17) so that $n_x = m$. In this case, the input matrix (18) is given by

$$G(x,z) = B_{\mathrm{u}} G_n(x,z) \tag{29}$$

so that $B = B_{\mathrm{u}}$. Now, for a given desired set point $(x_{\mathrm{e}}, z_{\mathrm{e}}) \in \mathbb{R}_+^{n_x} \times \overline{\mathbb{R}}_+^{n_z}$ and for given $\epsilon_1, \epsilon_2 > 0$, our aim is to design a control input $u(t)$, $t \geq 0$, such that $\|x(t) - x_{\mathrm{e}}\| < \epsilon_1$ and $\|z(t) - z_{\mathrm{e}}\| < \epsilon_2$ for all $t \geq T$, where $T \in [0,\infty)$, and $x(t) \geq\geq 0$ and $z(t) \geq\geq 0$ for all $t \geq 0$ and $(x_0, z_0) \in \overline{\mathbb{R}}_+^{n_x} \times \overline{\mathbb{R}}_+^{n_z}$. However, since in many applications of nonnegative systems and in particular, compartmental systems, it is often necessary to regulate a subset of the nonnegative state variables which usually include a central compartment, here we only require that $\|x(t) - x_{\mathrm{e}}\| < \epsilon_1$, $t \geq T$.

Theorem 4.1: Consider the nonlinear uncertain dynamical system $\mathcal{G}$ given by (16) and (17) where $f_x(\cdot,\cdot)$ and $G(\cdot,\cdot)$ are given by (19) and (29), respectively, $f_x(\cdot,\cdot)$ is essentially nonnegative with respect to x, $f_z(\cdot,\cdot)$ is essentially nonnegative with respect to z, and $\Delta f(\cdot,\cdot)$ is essentially nonnegative with respect to x and belongs to $\mathcal{F}$. For a given $x_{\mathrm{e}} \in \mathbb{R}_+^{n_x}$ assume there exist nonnegative vectors $z_{\mathrm{e}} \in \overline{\mathbb{R}}_+^{n_z}$ and $u_{\mathrm{e}} \in \overline{\mathbb{R}}_+^{n_x}$ such that (21) and (22) hold. Furthermore, assume that (17) is input-to-state stable at $z(t) \equiv z_{\mathrm{e}}$ with $x(t) - x_{\mathrm{e}}$ viewed as the input. Finally, let $K \in \mathbb{R}^{n_x \times n_x}$ be such that $-K$ is nonnegative and $A_{\mathrm{s}} \triangleq A + B_{\mathrm{u}} K$ is essentially nonnegative and asymptotically stable, and let q_i and γ_i, $i = 1, \ldots, n_x$, be positive constants. Then the neural adaptive feedback control law

$$u(t) = G_n^{-1}(x(t),z(t))\Big[K(x(t) - x_{\mathrm{e}}) - \hat{W}^{\mathrm{T}}(t)\hat{\sigma}(x(t),z(t))\Big] \tag{30}$$

where $\hat{W}^{\mathrm{T}}(t) \triangleq \text{block-diag}[\hat{W}_1^{\mathrm{T}}(t), \ldots, \hat{W}_{n_x}^{\mathrm{T}}(t)]$, $\hat{W}_i(t) \in \mathbb{R}^{s_i}$, $t \geq 0$, $i = 1, \ldots, n_x$, and $\hat{\sigma}(x,z) \triangleq [\hat{\sigma}_1^{\mathrm{T}}(x,z), \ldots, \hat{\sigma}_{n_x}^{\mathrm{T}}(x,z)]^{\mathrm{T}}$ with $\hat{\sigma}_i(x,z) = 0$ whenever $x_i = 0$, $i = 1, \ldots, n_x$, with update law

$$\begin{aligned}\dot{\hat{W}}_i(t) = q_i\Big[&(x_i(t) - x_{\mathrm{e}i})\hat{\sigma}_i(x(t),z(t)) \\ &- \gamma_i\|P^{1/2}(x(t) - x_{\mathrm{e}})\|\hat{W}_i(t)\Big] \quad \hat{W}_i(0) = \hat{W}_{i0} \\ &i = 1, \ldots, n_x\end{aligned} \tag{31}$$

where $P \triangleq \mathrm{diag}[p_1, \ldots, p_{n_x}] > 0$ satisfies

$$0 = A_{\mathrm{s}}^{\mathrm{T}} P + P A_{\mathrm{s}} + R \tag{32}$$

for a positive–definite matrix $R \in \mathbb{R}^{n_x \times n_x}$, guarantees that there exists a compact, positively invariant set $\mathcal{D}_\alpha \subset \overline{\mathbb{R}}_+^{n_x} \times \overline{\mathbb{R}}_+^{n_z} \times \mathbb{R}^{s \times n_x}$ such that $(x_{\mathrm{e}}, z_{\mathrm{e}}, W) \in \mathcal{D}_\alpha$, where $W \in \mathbb{R}^{s \times n_x}$, and the solution $(x(t), z(t), \hat{W}(t))$, $t \geq 0$, of the closed-loop system given by (16), (17), (30), and (31) is ultimately bounded for all $(x(0), z(0), \hat{W}(0)) \in \mathcal{D}_\alpha$ with ultimate bound $\|P^{1/2}(x(t) - x_{\mathrm{e}})\| < \varepsilon$, $t \geq T$, where

$$\varepsilon > \sqrt{\left(\frac{\nu}{\lambda_{\min}(RP^{-1})}\right)^2 + \sum_{i=1}^{n_x}\left(\frac{w_i^*}{2\sqrt{\hat{q}_i}} + \sqrt{\frac{\nu}{2q_i\gamma_i}}\right)^2} \tag{33}$$

$\hat{q}_i = q_i/p_i b_i$, and

$$\nu \triangleq 2\left[\sum_{i=1}^{n_x} p_i b_i^2(\varepsilon_i^* + \sqrt{s_i}w_i^*)^2\right]^{1/2} + \sum_{i=1}^{n_x}\frac{1}{2}p_i b_i \gamma_i w_i^{*2}. \tag{34}$$

Furthermore, $x(t) \geq\geq 0$ and $z(t) \geq\geq 0$ for all $t \geq 0$ and $(x_0, z_0) \in \overline{\mathbb{R}}_+^{n_x} \times \overline{\mathbb{R}}_+^{n_z}$.

Proof: The proof is given in the Appendix. □

Remark 4.1: In the case where the neural network approximation holds in $\mathbb{R}^{n_x} \times \mathbb{R}^{n_z}$, the assumptions $\tilde{\mathcal{D}}_\eta \subset \tilde{\mathcal{D}}_\alpha$ and $\mathcal{D}_z \subset \mathcal{D}_{\mathrm{c}z}$ invoked in the proof of Theorem 4.1 given in the Appendix are automatically satisfied. Furthermore, in this case the control law (30) ensures global ultimate boundedness of the error signals. However, the existence of a global neural network approximator for an uncertain nonlinear map cannot in general be established. Hence, as is common in the neural network literature, for a given arbitrarily large compact set $\mathcal{D}_{\mathrm{c}x} \times \mathcal{D}_{\mathrm{c}z} \subset \mathbb{R}^{n_x} \times \mathbb{R}^{n_z}$, we assume that there exists an approximator for the unknown nonlinear map up to a desired accuracy. This assumption ensures that in the error space $\tilde{\mathcal{D}}_{\mathrm{e}}$ (see the Appendix) there exists at least one Lyapunov level set such that $\tilde{\mathcal{D}}_\eta \subset \tilde{\mathcal{D}}_\alpha$. In the case where $\delta(\cdot,\cdot)$ is continuous on $\mathbb{R}^{n_x} \times \mathbb{R}^{n_z}$, it follows from the Stone-Weierstrass theorem that $\delta(\cdot,\cdot)$ can be approximated over an arbitrarily large compact set $\mathcal{D}_{\mathrm{c}x} \times \mathcal{D}_{\mathrm{c}z}$. In this case, our neuro adaptive controller guarantees semiglobal ultimate boundedness; that is, $\mathcal{D}_\alpha$ can be arbitrarily increased. An identical assumption is made in the proof of Theorem 5.1 given in the Appendix.

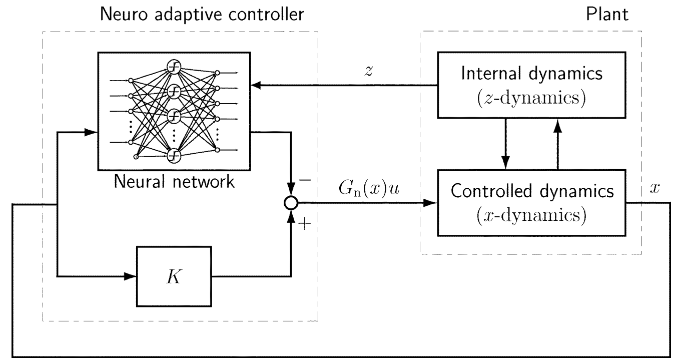

Fig. 1. Block diagram of the closed-loop system.

A block diagram showing the neuro adaptive control architecture given in Theorem 4.1 is shown in Fig. 1. It is important to note that the adaptive control law (30), (31) does not require the explicit knowledge of the optimal weighting matrix W and constants $\delta(x_{\mathrm{e}}, z_{\mathrm{e}})$ and u_{e}. All that is required is the existence of the nonnegative vectors z_{e} and u_{e} such that the equilibrium conditions (21) and (22) hold. Furthermore, in the case where $B_{\rm u}$ is an *unknown* positive diagonal matrix, we can take the gain matrix K to be diagonal so that $K = \operatorname{diag}[-k_1, \ldots, -k_{n_x}]$, where k_i, $i = 1, \ldots, n_x$, are positive. In this case, taking A in (19) to be the zero matrix, $A_{\rm s}$ is given by $A_{\rm s} = \operatorname{diag}[-b_1 k_1, \ldots, -b_{n_x} k_{n_x}]$ which is clearly essentially nonnegative and asymptotically stable. Furthermore, any $P = \operatorname{diag}[p_1, \ldots, p_{n_x}]$ satisfies (32). Finally, it is important to note that the control input signal $u(t)$, $t \geq 0$, in Theorem 4.1 can be negative depending on the values of $x(t)$, $z(t)$, and $\hat{W}(t)$, $t \geq 0$. However, as is required for nonnegative and compartmental dynamical systems the closed-loop plant states remain nonnegative.

Next, we generalize Theorem 4.1 to the case where the input matrix is not necessarily nonnegative. For this result $\operatorname{row}_i(K)$ denotes the ith row of $K \in \mathbb{R}^{n_x \times n_x}$.

Theorem 4.2: Consider the nonlinear uncertain dynamical system $\mathcal{G}$ given by (16) and (17) where $f_x(\cdot,\cdot)$ and $G(\cdot,\cdot)$ are given by (19) and (29), respectively, (with $G(x,z)$ not necessarily nonnegative) $f_x(\cdot,\cdot)$ is essentially nonnegative with respect to x, $f_z(\cdot,\cdot)$ is essentially nonnegative with respect to z, and $\Delta f(\cdot,\cdot)$ is essentially nonnegative with respect to x and belongs to $\mathcal{F}$. For a given $x_{\rm e} \in \mathbb{R}_+^{n_x}$ assume there exist a nonnegative vector $z_{\rm e} \in \overline{\mathbb{R}}_+^{n_z}$ and a vector $u_{\rm e} \in \mathbb{R}^{n_x}$ such that (21) and (22) hold with $f_x(x_{\rm e}, z_{\rm e}) \leq\leq 0$. Furthermore, assume that (17) is input-to-state stable at $z(t) \equiv z_{\rm e}$ with $x(t) - x_{\rm e}$ viewed as the input. Finally, let $K \in \mathbb{R}^{n_x \times n_x}$ be such that $(\operatorname{sgn} b_i)\operatorname{row}_i(K) \leq\leq 0$, $i = 1, \ldots, n_x$, and $A_{\rm s} \triangleq A + B_{\rm u} K$ is essentially nonnegative and asymptotically stable, and let q_i and γ_i, $i = 1, \ldots, n_x$, be positive constants. Then the neural adaptive feedback control law (30), where $\hat{W}^{\rm T}(t) \triangleq \text{block-diag}[\hat{W}_1^{\rm T}(t), \ldots, \hat{W}_{n_x}^{\rm T}(t)]$, $\hat{W}_i(t) \in \mathbb{R}^{s_i}$, $t \geq 0, i = 1, \ldots, n_x$, and $\hat{\sigma}(x,z) \triangleq [\hat{\sigma}_1^{\rm T}(x,z), \ldots, \hat{\sigma}_{n_x}^{\rm T}(x,z)]^{\rm T}$ with $\hat{\sigma}_i(x,z) = 0$ whenever $x_i = 0, i = 1, \ldots, n_x$, with update law

$$\begin{aligned}\dot{\hat{W}}_i(t) = &(\operatorname{sgn} b_i) q_i \Big[(x_i(t) - x_{{\rm e}i})\hat{\sigma}_i(x(t), z(t)) \\ &- \gamma_i \|P^{1/2}(x(t) - x_{\rm e})\| \hat{W}_i(t)\Big] \quad \hat{W}_i(0) = \hat{W}_{i0} \\ &i = 1, \ldots, n_x \end{aligned} \tag{35}$$

where $P \triangleq \operatorname{diag}[p_1, \ldots, p_{n_x}] > 0$ satisfies (32), guarantees that there exists a compact, positively invariant set $\mathcal{D}_\alpha \subset \overline{\mathbb{R}}_+^{n_x} \times \overline{\mathbb{R}}_+^{n_z} \times \mathbb{R}^{s \times n_x}$ such that $(x_{\rm e}, z_{\rm e}, W) \in \mathcal{D}_\alpha$, where $W \in \mathbb{R}^{s \times n_x}$, and the solution $(x(t), z(t), \hat{W}(t))$, $t \geq 0$, of the closed-loop system given by (16), (17), (30), and (35) is ultimately bounded for all $(x(0), z(0), \hat{W}(0)) \in \mathcal{D}_\alpha$ with ultimate bound $\|P^{1/2}(x(t) - x_{\rm e})\| < \varepsilon$, $t \geq T$, where ε is given by (33). Furthermore, $x(t) \geq\geq 0$ and $z(t) \geq\geq 0$ for all $t \geq 0$ and $(x_0, z_0) \in \overline{\mathbb{R}}_+^{n_x} \times \overline{\mathbb{R}}_+^{n_z}$.

Proof: The proof is identical to the proof of Theorem 4.1 given in the Appendix with Q replaced by $Q = \operatorname{diag}\Big[q_1/(p_1|b_1|), \ldots, q_{n_x}/(p_{n_x}|b_{n_x}|)\Big]$. □

Finally, in the case where $B_{\rm u}$ is an *unknown* diagonal matrix but the sign of each diagonal element is known, we can take the gain matrix K to be diagonal so that $K = \operatorname{diag}[k_1, \ldots, k_{n_x}]$, where k_i is such that $(\operatorname{sgn} b_i) k_i < 0$, $i = 1, \ldots, n_x$. In this case, taking A in (19) to be the zero matrix, $A_{\rm s}$ is given by $A_{\rm s} = \operatorname{diag}[b_1 k_1, \ldots, b_{n_x} k_{n_x}]$ which is essentially nonnegative and asymptotically stable.

V. Neural Adaptive Control for Nonlinear Nonnegative Uncertain Systems With Nonnegative Control

As discussed in the Introduction, control (source) inputs of drug delivery systems for physiological and pharmacological processes are usually constrained to be nonnegative as are the system states. Hence, in this section, we develop neuro adaptive control laws for nonnegative systems with nonnegative control inputs. Specifically, for a given desired set point $(x_{\rm e}, z_{\rm e}) \in \mathbb{R}_+^{n_x} \times \mathbb{R}_+^{n_z}$ and for given $\epsilon_1, \epsilon_2 > 0$, our aim is to design a nonnegative control input $u(t)$, $t \geq 0$, such that $\|x(t) - x_{\rm e}\| < \epsilon_1$ and $\|z(t) - z_{\rm e}\| < \epsilon_2$ for all $t \geq T$, where $T \in [0, \infty)$, and $x(t) \geq\geq 0$ and $z(t) \geq\geq 0$ for all $t \geq 0$ and $(x_0, z_0) \in \mathbb{R}_+^{n_x} \times \mathbb{R}_+^{n_z}$. However, since in many applications of nonnegative systems and in particular, compartmental systems, it is often necessary to regulate a subset of the nonnegative state variables which usually include a central compartment, here we only require that $\|x(t) - x_{\rm e}\| < \epsilon_1, t \geq T$. Furthermore, we assume that we have m *independent* control inputs such that the input matrix function is given by $G(x,z) = \operatorname{diag}[g_1(x,z), \ldots, g_m(x,z)]$, where $g_i : \mathbb{R}^{n_x} \times \mathbb{R}^{n_z} \to \mathbb{R}_+, i = 1, \ldots, m$. For compartmental systems this assumption is not restrictive since control inputs correspond to control inflows to each individual compartment.

Theorem 5.1: Consider the nonlinear uncertain dynamical system $\mathcal{G}$ given by (16) and (17) where $f_x(\cdot,\cdot)$ and $G(\cdot,\cdot)$ are given by (19) and (29), respectively, A is essentially nonnegative and asymptotically stable, $f_x(\cdot,\cdot)$ is essentially nonnegative with respect to x, $f_z(\cdot,\cdot)$ is essentially nonnegative with respect to z, and $\Delta f(\cdot,\cdot)$ is essentially nonnegative with respect to x and belongs to $\mathcal{F}$. For a given $x_{\rm e} \in \mathbb{R}_+^{n_x}$ assume there exist positive vectors $z_{\rm e} \in \mathbb{R}_+^{n_z}$ and $u_{\rm e} \in \mathbb{R}_+^{n_x}$ such that (21) and (22) hold and the equilibrium point $(x_{\rm e}, z_{\rm e}) \in \mathbb{R}_+^{n_x} \times \mathbb{R}_+^{n_z}$ of (16), (17) is globally asymptotically stable with $u(t) \equiv u_{\rm e}$. Furthermore, assume that (17) is input-to-state stable at $z(t) \equiv z_{\rm e}$ with $x(t) - x_{\rm e}$ viewed as the input. Finally, let q_i and γ_i, $i = 1, \ldots, n_x$, be positive constants and k_i, $i = 1, \ldots, n_x$, be nonpositive constants. Then the neural adaptive feedback control law

$$u_i(t) = \max\{0, \hat{u}_i(t)\}, \quad i = 1, \ldots, n_x \tag{36}$$

where

$$\begin{aligned}\hat{u}_i(t) = &g_{n_i}^{-1}(x(t), z(t))\Big[k_i(x_i(t) - x_{{\rm d}i}) \\ &- \hat{W}_i^{\rm T}(t)\sigma_i(x(t), z(t))\Big] \quad i = 1, \ldots, n_x\end{aligned} \tag{37}$$

and $\hat{W}_i(t) \in \mathbb{R}^{s_i}$, $t \geq 0$, $i = 1, \ldots, n_x$, with update law

$$\begin{aligned}\dot{\hat{W}}_i(t) = &q_i \Big[(x_i(t) - x_{{\rm e}i})\sigma_i(x(t), z(t)) \\ &- \gamma_i \|P^{1/2}(x(t) - x_{\rm e})\| \hat{W}_i(t)\Big] \quad \hat{W}_i(0) = \hat{W}_{i0} \\ &i = 1, \ldots, n_x\end{aligned} \tag{38}$$

where $P \triangleq \operatorname{diag}[p_1, \ldots, p_{n_x}] > 0$ satisfies

$$0 = A^{\rm T} P + PA + R \tag{39}$$

for a positive–definite matrix $R \in \mathbb{R}^{n_x \times n_x}$, guarantees that there exists a compact, positively invariant set $\mathcal{D}_\alpha \subset \overline{\mathbb{R}}_+^{n_x} \times \overline{\mathbb{R}}_+^{n_z} \times \mathbb{R}^{s \times n_x}$ such that $(x_{\rm e}, z_{\rm e}, W) \in \mathcal{D}_\alpha$, where $W \in \mathbb{R}^{s \times n_x}$, and the solution $(x(t), z(t), \hat{W}(t)), t \geq 0$, of the closed-loop system given by (16), (17), (36), and (38) is ultimately bounded for all $(x(0), z(0), \hat{W}(0)) \in \mathcal{D}_\alpha$ with ultimate bound $\|P^{1/2}(x(t) - x_{\rm e})\| < \varepsilon, t \geq T$, where

$$\varepsilon > \left[\left(\frac{\nu}{\lambda_{\min}(RP^{-1})} \right)^2 + \sum_{i=1}^{n_x} \left[\frac{1}{2} \left(\frac{\sqrt{b_i s_i}}{q_i \gamma_i^2} + \frac{w_i^*}{\sqrt{\hat{q}_i}} \right) + \sqrt{\frac{\nu}{2 q_i \gamma_i}} \right]^2 \right]^{1/2} \quad (40)$$

$\hat{q}_i = q_i / p_i b_i$, and

$$\nu \triangleq \left(\sum_{i=1}^{n_x} p_i b_i^2 \varepsilon_i^{*2} + \sum_{i=1}^{n_x} \left[2 b_i \sqrt{p_i s_i} w_i^* + \frac{q_i \gamma_i}{2} \left(\sqrt{\frac{b_i s_i}{q_i \gamma_i^2}} + \frac{w_i^*}{\sqrt{\hat{q}_i}} \right)^2 \right] \right)^{1/2}. \quad (41)$$

Furthermore, $u(t) \geq\geq 0$, $x(t) \geq\geq 0$, and $z(t) \geq\geq 0$ for all $t \geq 0$ and $(x_0, z_0) \in \overline{\mathbb{R}}_+^{n_x} \times \overline{\mathbb{R}}_+^{n_z}$.

Proof: The proof is given in the Appendix. □

In Theorem 5.1 we assumed that the equilibrium point $(x_{\rm e}, z_{\rm e})$ of (16), (17) is globally asymptotically stable with $u(t) \equiv u_{\rm e}$. In general, however, unlike linear nonnegative systems with asymptotically stable plant dynamics, a given set point $(x_{\rm e}, z_{\rm e}) \in \overline{\mathbb{R}}_+^{n_x} \times \overline{\mathbb{R}}_+^{n_z}$ for the nonlinear nonnegative dynamical system (16), (17) may not be asymptotically stabilizable with a constant control $u(t) \equiv u_{\rm e} \in \overline{\mathbb{R}}_+^{n_x}$. However, if $f(\tilde{x}) \triangleq [f_x^{\rm T}(x,z), f_z^{\rm T}(x,z)]^{\rm T}$, where $\tilde{x} \triangleq [x^{\rm T}, z^{\rm T}]^{\rm T}$, is homogeneous, cooperative; that is, the Jacobian matrix $\partial f(\tilde{x})/\partial \tilde{x}$ is essentially nonnegative for all $\tilde{x} \in \overline{\mathbb{R}}_+^{n_x+n_z}$ [33], the Jacobian matrix $\partial f(\tilde{x})/\partial \tilde{x}$ is irreducible for all $\tilde{x} \in \overline{\mathbb{R}}_+^{n_x+n_z}$ [33], and the zero solution $\tilde{x}(t) \equiv 0$ of the undisturbed $(u(t) \equiv 0)$ system (16), (17) is globally asymptotically stable, then the set point $(x_{\rm e}, z_{\rm e}) \in \overline{\mathbb{R}}_+^{n_x} \times \overline{\mathbb{R}}_+^{n_z}$ satisfying (21), (22) is a unique equilibrium point with $u(t) \equiv u_{\rm e} \in \mathbb{R}_+^{n_x}$ and is also asymptotically stable for all $(x_0, z_0) \in \overline{\mathbb{R}}_+^{n_x} \times \overline{\mathbb{R}}_+^{n_z}$ [34]. This implies that the solution $(x(t), z(t)) \equiv (x_{\rm e}, z_{\rm e})$ to (16), (17) with $u(t) \equiv u_{\rm e}$ is asymptotically stable for all $(x_0, z_0) \in \overline{\mathbb{R}}_+^{n_x} \times \overline{\mathbb{R}}_+^{n_z}$.

It is important to note that unlike Theorem 4.1, Theorem 5.1 does not require that the set of basis functions $\sigma_i(\cdot,\cdot)$, $i = 1, \ldots, n_x$, be essentially nonnegative nor satisfy $\sigma_i(x,z) = 0$ whenever $x_i = 0, i = 1, \ldots, n_x$. This is due to the fact that the control input is constrained to be nonnegative and, hence, the neuro adaptive controller given by Theorem 5.1 cannot destroy nonnegativity of the closed-loop plant states.

VI. Neural Adaptive Control for Continuous Stirred Tank Reactors

In this section, we apply the proposed neuro adaptive control framework to temperature regulation of chemical reactors. In particular, we consider a perfectly mixed, continuously stirred tank reactor shown in Fig. 2 involving a single, first-order exothermic (i.e., energy releasing) irreversible reaction $\mathrm{A} \rightarrow \mathrm{B}$. The model involves fluid streams that are continuously fed and removed from the reactor. Since we assume perfect mixing in the reactor, the exit stream has the same concentration and temperature as the reactor fluid. Furthermore, the jacket surrounding the reactor is assumed to be perfectly mixed and at a lower temperature than the reactor. In this case, energy (in the form of heat) transfers through the reactor walls into the jacket, removing the heat generated by the reaction. A mass and energy balance of the reactor, assuming constant volume, heat capacity, and density, yields [35]–[38]

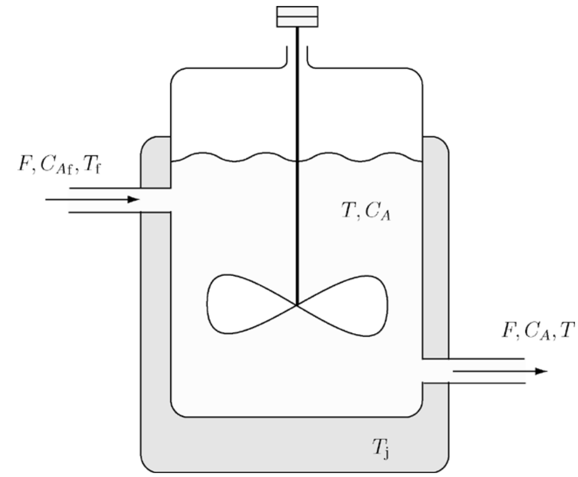

Fig. 2. Exothermic continuously stirred tank reactor.

$$\dot{C}_{\rm A}(t) = \frac{F}{\hat{V}}(C_{\rm Af} - C_{\rm A}(t)) - r(T(t), C_{\rm A}(t)), \quad C_{\rm A}(0) = C_{\rm A0}, \quad t \geq 0 \quad (42)$$

$$\dot{T}(t) = \frac{F}{\hat{V}}(T_{\rm f} - T(t)) - \left(\frac{-\Delta H}{\rho c_p} \right) r(T(t), C_{\rm A}(t)) - \frac{UA}{\hat{V} \rho c_p}(T(t) - T_{\rm j}(t)), \quad T(0) = T_0 \quad (43)$$

where $C_{\rm A}(\cdot)$ is the concentration of reactant A in the reactor effluent in mols/liter, $C_{\rm Af}$ is the concentration of reactant A in the feed stream in mols/liter, $T(\cdot)$ is the reactor temperature in degrees Kelvin, $T_{\rm j}(\cdot)$ is the jacket temperature in degrees Kelvin, $T_{\rm f}$ is the feed temperature in degrees Kelvin, F is the constant feed flow rate in liters/min, $\hat{V}$ is the reactor volume in liters, $-\Delta H$ is the heat of reaction in Joules/mol, ρ is the density in grams/liter, c_p is the specific heat in $\mathrm{Joules/(gram \cdot Kelvin)}$, UA is the heat transfer term in $\mathrm{Joules/(min \cdot Kelvin)}$, and $r(T, C_{\rm A})$ is the rate of reaction satisfying Arrhenius' law given by

$$r(T, C_{\rm A}) = k_0 C_{\rm A} e^{-\Delta E/RT} \quad (44)$$

where k_0 is the rate constant in $\min^{-1}$, ΔE is the activation energy in Joules/mol, and R is the ideal gas constant in $\mathrm{Joules/(mol \cdot Kelvin)}$.

Due to the exponential nonlinearity in $r(T, C_{\rm A})$, the nonlinear kinetic (42), (43) can exhibit multiple equilibria, limit cycles, and chaos for fixed jacket temperatures. Here, our control objective is to regulate the reactor temperature $T(\cdot)$ to a prescribed set point $T_{\rm e}$ by controlling the jacket temperature $T_{\rm j}(\cdot)$. Note that with $x = T$, $z = C_{\rm A}$, and $u = T_{\rm j}$, (42) and (43) can be written in state-space form (16) and (17) with

$$f_x(x,z) = -(a_1 + a_3)x + a_4 r(x,z) + a_1 d \quad (45)$$

$$f_z(x,z) = -a_1 z - r(x,z) + a_2 \quad (46)$$

$$G(x,z) = b \quad (47)$$

TABLE I
SYSTEM PARAMETER VALUES [39]

Variable	Value
UA	5×10^4 J/min K
$C_{\rm A}$	0.5 mol/ℓ
$C_{\rm Af}$	1 mol/ℓ
c_p	0.239 J/gK
F	100 ℓ/min
k_0	7.2×10^{10} min^{-1}
T	350 K
$T_{\rm f}$	350 K
$\hat{V}$	100 ℓ
$\Delta E/R$	8750 K
$(-\Delta H)$	5×10^4 J/mol
ρ	1000 g/ℓ

where $a_1 = F/\hat{V}$, $a_2 = (F/\hat{V})C_{\rm Af}$, $a_3 = b = UA/\hat{V}\rho c_p$, $a_4 = \Delta H/\rho c_p$, and $d = T_{\rm f}$. Note that $f_x(x,z)$ and $f_z(x,z)$ are essentially nonnegative with respect to x and z, respectively and, hence, it follows from [12, Prop. 7.1] that the state trajectory of (42) and (43) remain in the nonnegative orthant of the state–space for nonnegative initial conditions and a nonnegative input. We assume that there exists an equilibrium point $(x_{\rm e}, z_{\rm e}) \in \overline{\mathbb{R}}_+ \times \overline{\mathbb{R}}_+$ so that (21) and (22) are satisfied [37]. Furthermore, we assume that the system kinetics are uncertain with respect to the temperature as well as a_1, a_2, a_3, a_4, b, and d are uncertain parameters.

To see that (43) is input-to-state stable with $T(\cdot)$ viewed as the input, define $e_x(t) \triangleq x(t) - x_{\rm e}$ and $e_z(t) \triangleq z(t) - z_{\rm e}$ so that $\tilde{f}_z(e_x, e_z) \triangleq f_z(e_x + x_{\rm e}, e_z + z_{\rm e}) - f_z(x_{\rm e}, z_{\rm e})$ is given by

$$\tilde{f}_z(e_x, e_z) = -a_1 e_z - k_0 e^{-\Delta E/R({\rm e}_x + x_{\rm e})} e_z - k_0 z_{\rm e}\left(e^{-\Delta E/R({\rm e}_x + x_{\rm e})} - e^{-\Delta E/R x_{\rm e}}\right). \quad (48)$$

Now, defining $V_z(e_z) \triangleq (1/2)e_z^2$ and noting that $\left(e^{-\Delta E/R({\rm e}_2 + x_{{\rm e}2})} - e^{-\Delta E/R x_{{\rm e}2}}\right)$ is bounded, it follows that

$$\begin{aligned} &V_z'(e_z)\tilde{f}_z(e_x, e_z) \\ &\leq -a_1 e_z^2 - k_0 z_{\rm e}\left(e^{-\Delta E/R({\rm e}_x + x_{\rm e})} - e^{-\Delta E/R x_{\rm e}}\right) e_z \\ &\leq -\|e_z\|\left[a_1\|e_z\| - k_0 z_{\rm e}\left\|e^{-\Delta E/R({\rm e}_x + x_{\rm e})} - e^{-\Delta E/R x_{\rm e}}\right\|\right] \\ &\leq 0, \quad \|e_z\| \geq \frac{k_0 z_{\rm e}}{a_1}\left\|e^{-\Delta E/R({\rm e}_x + x_{\rm e})} - e^{-\Delta E/R x_{\rm e}}\right\| \end{aligned} \quad (49)$$

which shows that $\dot{e}_z(t) = \tilde{f}_z(e_x(t), e_z(t))$, $t \geq 0$, is input-to-state stable with e_x viewed as the input. Hence, it follows from Theorem 5.1 that the adaptive feedback controller (36) with update law (38) guarantees that the closed-loop system is ultimately bounded and, hence, there exist positive constants ε and T such that $|T(t) - T_{\rm e}| < \varepsilon$, $t \geq T$, for all (uncertain) positive system parameters $a_1, \ldots, a_4$, b, d, and all (uncertain) continuous rate of reaction $r(\cdot,\cdot)$.

For our simulation, we choose the system parameters given in Table I. With $T_{\rm e} = 375$ K, $k_1 = -14$

$$\sigma_1(x,z) = \left[\frac{1}{1+e^{-a(x-{\rm T_e})}}, \ldots, \frac{1}{1+e^{-6a(x-{\rm T_e})}}\right. \left.\frac{1}{1+e^{-a(z-0.5)}}, \ldots, \frac{1}{1+e^{-6a(z-0.5)}}\right]^{\rm T}$$

$a = 0.5$, $q_1 = 20$, $\gamma_1 = 0.01$, and initial conditions $C_{\rm A}(0) = 0.5$ mol/ℓ, $T(0) = 350$ K, and $\hat{W}(0) = 0$ K, Fig. 3 shows the state trajectories (i.e., reactor temperature and concentration of reactant A) versus time and the control signal (i.e., jacket temperature) versus time. Finally, Fig. 4 shows the neural network weight history versus time.

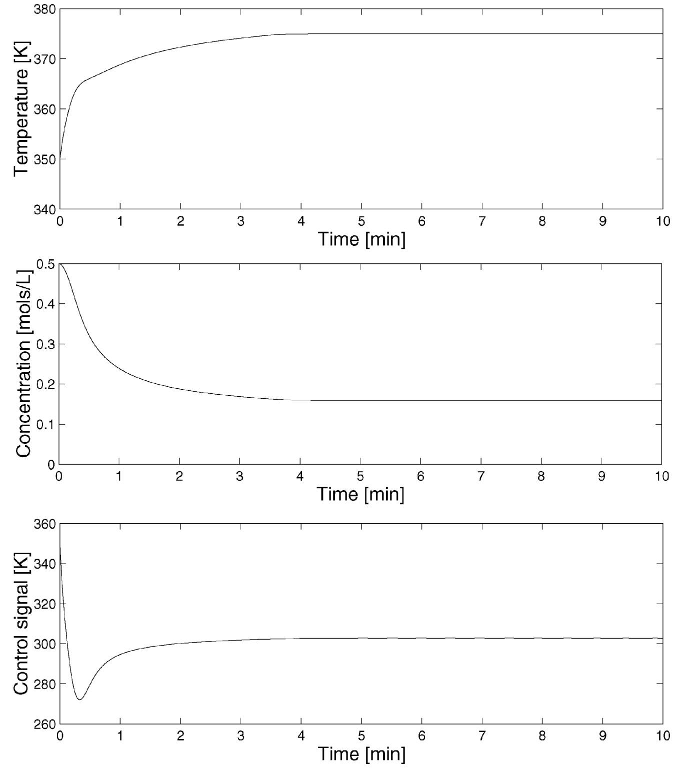

Fig. 3. State trajectories (reactor temperature and concentration of reactant A) and control signal (jacket temperature) versus time.

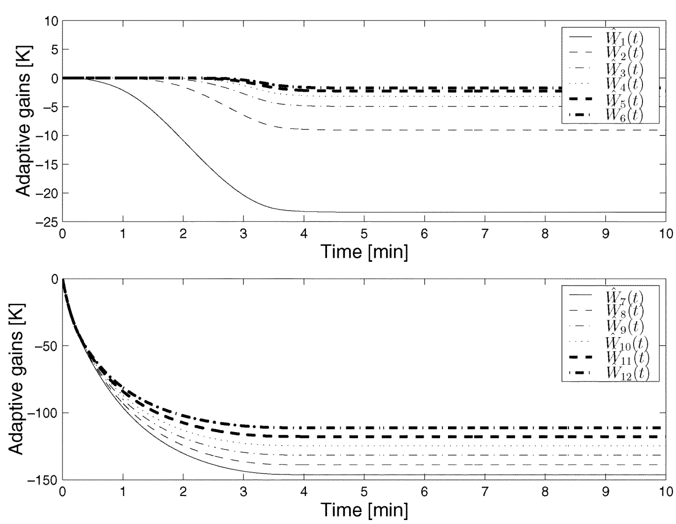

Fig. 4. Neural network weighting functions versus time.

VII. CONCLUSION

Nonnegative and compartmental systems are widely used to capture system dynamics involving the interchange of mass and energy between homogenous subsystems or compartments. In this paper, we developed a neural adaptive control framework for adaptive set-point regulation of nonlinear uncertain nonnegative and compartmental systems. Using Lyapunov-like methods the proposed framework was shown to guarantee ultimate boundedness of the error signals corresponding to the physical system states and the neural network weighting gains while additionally guaranteeing the nonnegativity of the closed-loop system states associated with the plant dynamics. We then generalized our neuro adaptive controller to address the problem of nonnegative systems with nonnegative control inputs. This generalization is crucial for physiological, pharmacological, and chemical processes as control inputs are usually constrained to be nonnegative.

APPENDIX

To prove Theorem 4.1, note that with $u(t)$, $t \geq 0$, given by (30) it follows from (16), (19), and (29) that:

$$\begin{aligned}\dot{x}(t) = {} & Ax(t) + \Delta f(x(t), z(t)) + B_{\mathrm{u}} K(x(t) - x_{\mathrm{e}}) \\ & - B_{\mathrm{u}} \hat{W}^{\mathrm{T}}(t)\hat{\sigma}(x(t), z(t)), \quad x(0) = x_0, \quad t \geq 0. \end{aligned} \tag{50}$$

Now, defining $e_x(t) \triangleq x(t) - x_{\mathrm{e}}$ and $e_z(t) \triangleq z(t) - z_{\mathrm{e}}$, using (20)–(22), and noting that $A_{\mathrm{s}} = A + B_{\mathrm{u}} K$, it follows from (17) and (50) that:

$$\begin{aligned}\dot{e}_x(t) &= A_{\mathrm{s}} e_x(t) + A x_{\mathrm{e}} + \Delta f(x(t), z(t)) \\ &\quad - B_{\mathrm{u}} \hat{W}^{\mathrm{T}}(t) \hat{\sigma}(x(t), z(t)) \\ &= A_{\mathrm{s}} e_x(t) + B_{\mathrm{u}}[\delta(x(t), z(t)) - \delta(x_{\mathrm{e}}, z_{\mathrm{e}}) \\ &\quad - G_n(x_{\mathrm{e}}, z_{\mathrm{e}}) u_{\mathrm{e}} - \hat{W}^{\mathrm{T}}(t)\sigma(x(t), z(t))] \\ &\quad + B_{\mathrm{u}} \hat{W}^{\mathrm{T}}(t)[\sigma(x(t), z(t)) - \hat{\sigma}(x(t), z(t))], \\ e_x(0) &= x_0 - x_{\mathrm{e}}, \quad t \geq 0 \end{aligned} \tag{51}$$

and

$$\dot{e}_z(t) = \tilde{f}_z(e_x(t), e_z(t)), \quad e_z(0) = z_0 - z_{\mathrm{e}} \tag{52}$$

where $\tilde{f}_z(e_x, e_z) \triangleq f_z(e_x + x_{\mathrm{e}}, e_z + z_{\mathrm{e}}) - f_z(x_{\mathrm{e}}, z_{\mathrm{e}})$ and $\sigma(x, z) \triangleq [\sigma_1^{\mathrm{T}}(x, z), \ldots, \sigma_{n_x}^{\mathrm{T}}(x, z)]^{\mathrm{T}}$ is a basis function satisfying (24). Furthermore, since A_{s} is essentially nonnegative and asymptotically stable, it follows from [12. Th. 3.3] that there exist a positive *diagonal* matrix $P = \mathrm{diag}[p_1, \ldots, p_{n_x}]$ and a positive–definite matrix $R \in \mathbb{R}^{n_x \times n_x}$ such that (32) holds.

Next, to show ultimate boundedness of the closed-loop system (31), (51), and (52) consider the Lyapunov-like function

$$V(e_x, e_z, \tilde{W}) = e_x^{\mathrm{T}} P e_x + \mathrm{tr}\, \tilde{W} Q^{-1} \tilde{W}^{\mathrm{T}} \tag{53}$$

where $Q \triangleq \mathrm{diag}\left[\hat{q}_1, \ldots, \hat{q}_{n_x}\right] = \mathrm{diag}\left[q_1/p_1 b_1, \ldots, q_{n_x}/p_{n_x} b_{n_x}\right]$, $\tilde{W}(t) \triangleq \hat{W}(t) - W$, and $W^{\mathrm{T}} \triangleq \text{block-diag}[W_1^{\mathrm{T}}, \ldots, W_{n_x}^{\mathrm{T}}]$. Note that (53) satisfies (9) with $x_1 = [e_x^{\mathrm{T}}, \tilde{W}_1^{\mathrm{T}} \ldots, \tilde{W}_{n_x}^{\mathrm{T}}]^{\mathrm{T}}$, $x_2 = e_z$, $\alpha(\|x_1\|) = \beta(\|x_1\|) = \|x_1\|^2$, where $\|x_1\|^2 \triangleq e_x^{\mathrm{T}} P e_x + \mathrm{tr}\, \tilde{W} Q^{-1} \tilde{W}^{\mathrm{T}}$. Furthermore, $\alpha(\|x_1\|)$ is a class $\mathcal{K}_\infty$ function. Now, letting $e_x(t)$, $t \geq 0$, denote the solution to (51) and using (24), (25), and (31), it follows that the time derivative of $V(e_x, e_z, \tilde{W})$ along the closed-loop system trajectories is given by (54), shown at the bottom of the page. Next, completing squares yields

$$\begin{aligned}\dot{V}&(e_x(t), e_z(t), \tilde{W}(t)) \\ &\leq -e_x^{\mathrm{T}}(t) R e_x(t) + 2\|P^{1/2} e_x(t)\| \\ &\quad \cdot \left(\sum_{i=1}^{n_x} p_i b_i^2 (\varepsilon_i^* + \sqrt{s_i} w_i^*)^2\right)^{1/2} \\ &\quad - \sum_{i=1}^{n_x} 2 p_i b_i \gamma_i \|P^{1/2} e_x(t)\| \tilde{W}_i^{\mathrm{T}}(t) \tilde{W}_i(t) \\ &\quad - \sum_{i=1}^{n_x} 2 p_i b_i \gamma_i \|P^{1/2} e_x(t)\| \tilde{W}_i^{\mathrm{T}}(t) W_i \\ &\leq -\lambda_{\min}(R P^{-1}) \|P^{1/2} e_x(t)\|^2 \\ &\quad + 2 \left(\sum_{i=1}^{n_x} p_i b_i^2 (\varepsilon_i^* + \sqrt{s_i} w_i^*)^2\right)^{1/2} \|P^{1/2} e_x(t)\| \\ &\quad - \sum_{i=1}^{n_x} 2 p_i b_i \hat{q}_i \gamma_i \|P^{1/2} e_x(t)\| \|\hat{q}_i^{-1/2} \tilde{W}_i(t)\|^2 \end{aligned}$$

$$\begin{aligned}&\dot{V}(e_x(t), e_z(t), \tilde{W}(t)) \\ &= 2 e_x^{\mathrm{T}}(t) P \Big[A_{\mathrm{s}} e_x(t) + B_{\mathrm{u}}[\delta(x(t), z(t)) - \delta(x_{\mathrm{e}}, z_{\mathrm{e}}) \\ &\quad - G_n(x_{\mathrm{e}}, z_{\mathrm{e}}) u_{\mathrm{e}} - \hat{W}^{\mathrm{T}}(t) \sigma(x(t), z(t))] + B_{\mathrm{u}} \hat{W}^{\mathrm{T}}(t)[\sigma(x(t), z(t)) - \hat{\sigma}(x(t), z(t))]\Big] + 2\mathrm{tr}\, \tilde{W}^{\mathrm{T}}(t) Q^{-1} \dot{\hat{W}}(t) \\ &= -e_x^{\mathrm{T}}(t) R e_x(t) + \sum_{i=1}^{n_x} 2 p_i b_i e_{xi}(t) \Big[-\tilde{W}_i^{\mathrm{T}}(t) \sigma_i(x(t), z(t)) + \varepsilon_i(x(t), z(t))\Big] \\ &\quad + \sum_{i=1}^{n_x} 2 p_i b_i e_{xi}(t) \hat{W}_i^{\mathrm{T}}(t)[\sigma_i(x(t), z(t)) - \hat{\sigma}_i(x(t), z(t))] \\ &\quad + \sum_{i=1}^{n_x} 2 p_i b_i \tilde{W}_i^{\mathrm{T}}(t) \Big[e_{xi}(t) \hat{\sigma}_i(x(t), z(t)) - \gamma_i \|P^{1/2}(x(t) - x_{\mathrm{e}})\| \hat{W}_i(t)\Big] \\ &= -e_x^{\mathrm{T}}(t) R e_x(t) + \sum_{i=1}^{n_x} 2 p_i b_i \varepsilon_i(x(t), z(t)) e_{xi}(t) + \sum_{i=1}^{n_x} 2 p_i b_i e_{xi}(t) W_i^{\mathrm{T}}[\sigma_i(x(t), z(t)) - \hat{\sigma}_i(x(t), z(t))] \\ &\quad - \sum_{i=1}^{n_x} 2 p_i b_i \gamma_i \|P^{1/2} e_x(t)\| \tilde{W}_i^{\mathrm{T}}(t) \hat{W}_i(t). \end{aligned} \tag{54}$$

$$
\begin{aligned}
&\quad + \sum_{i=1}^{n_x} 2p_i b_i \sqrt{\hat{q}_i}\gamma_i w_i^* \|P^{1/2}e_x(t)\| \|\hat{q}_i^{-1/2}\tilde{W}_i(t)\| \\
&= \|P^{1/2}e_x(t)\| \Big[-\lambda_{\min}(RP^{-1}) \|P^{1/2}e_x(t)\| \\
&\quad + 2\left(\sum_{i=1}^{n_x} p_i b_i^2 (\varepsilon_i^* + \sqrt{s_i} w_i^*)^2\right)^{1/2} \\
&\quad - \sum_{i=1}^{n_x} 2q_i\gamma_i \|\hat{q}_i^{-1/2}\tilde{W}_i(t)\|^2 \\
&\quad + \sum_{i=1}^{n_x} 2q_i \hat{q}_i^{-1/2}\gamma_i w_i^* \|\hat{q}_i^{-1/2}\tilde{W}_i(t)\| \Big] \\
&= \|P^{1/2}e_x(t)\| \Big[-\lambda_{\min}(RP^{-1}) \|P^{1/2}e_x(t)\| \\
&\quad + 2\left(\sum_{i=1}^{n_x} p_i b_i^2 (\varepsilon_i^* + \sqrt{s_i} w_i^*)^2\right)^{1/2} \\
&\quad - \sum_{i=1}^{n_x} 2q_i\gamma_i \Big[\|\hat{q}_i^{-1/2}\tilde{W}_i(t)\| - \frac{w_i^*}{2\sqrt{\hat{q}_i}} \Big]^2 \\
&\quad + \sum_{i=1}^{n_x} \frac{1}{2} p_i b_i \gamma_i {w_i^*}^2 \Big] \\
&\le \|P^{1/2}e_x(t)\| \Big[-\lambda_{\min}(RP^{-1}) \|P^{1/2}e_x(t)\| \\
&\quad - \sum_{i=1}^{n_x} 2q_i\gamma_i \Big[\|\hat{q}_i^{-1/2}\tilde{W}_i(t)\| - \frac{w_i^*}{2\sqrt{\hat{q}_i}} \Big]^2 + \nu \Big]
\end{aligned} \tag{55}
$$

where ν is given by (34). Now, for

$$\|P^{1/2}e_x\| \ge \frac{\nu}{\lambda_{\min}(RP^{-1})} \triangleq \alpha_x \tag{56}$$

or

$$\|\hat{q}_i^{-1/2}\tilde{W}_i\| \ge \frac{w_i^*}{2\sqrt{\hat{q}_i}} + \sqrt{\frac{\nu}{2q_i\gamma_i}} \triangleq \alpha_{\tilde{W}_i}, \ i=1,\ldots,n_x \tag{57}$$

it follows that $\dot{V}(e_x(t), e_z(t), \tilde{W}(t)) \le 0$ for all $t \ge 0$; that is, $\dot{V}(e_x(t), e_z(t), \tilde{W}(t)) \le 0$ for all $(e_x(t), e_z(t), \tilde{W}(t)) \in \tilde{\mathcal{D}}_{\rm e} \backslash \tilde{\mathcal{D}}_{\rm r}$ and $t \ge 0$, where

$$\tilde{\mathcal{D}}_{\rm e} \triangleq \left\{ (e_x, e_z, \tilde{W}) \in \mathbb{R}^{n_x} \times \mathbb{R}^{n_z} \times \mathbb{R}^{s \times n_x} : x \in \mathcal{D}_{{\rm c}x} \right\} \tag{58}$$

$$\tilde{\mathcal{D}}_{\rm r} \triangleq \Big\{ (e_x, e_z, \tilde{W}) \in \mathbb{R}^{n_x} \times \mathbb{R}^{n_z} \times \mathbb{R}^{s \times n_x} : \|P^{1/2 e_x}\| \le \alpha_x, \|\hat{q}_i^{-1/2}\tilde{W}_i\| \le \alpha_{\tilde{W}_i}, \ i=1,\ldots,n_x \Big\}. \tag{59}$$

Next, define

$$\tilde{\mathcal{D}}_\alpha \triangleq \Big\{ (e_x, e_z, \tilde{W}) \in \mathbb{R}^{n_x} \times \mathbb{R}^{n_z} \times \mathbb{R}^{s \times n_x} : V(e_x, e_z, \tilde{W}) \le \alpha \Big\} \tag{60}$$

where α is the maximum value such that $\tilde{\mathcal{D}}_\alpha \subseteq \tilde{\mathcal{D}}_{\rm e}$, and define

$$\tilde{\mathcal{D}}_\eta \triangleq \Big\{ (e_x, e_z, \tilde{W}) \in \mathbb{R}^{n_x} \times \mathbb{R}^{n_z} \times \mathbb{R}^{s \times n_x} : V(e_x, e_z, \tilde{W}) \le \eta \Big\} \tag{61}$$

where

$$
\begin{aligned}
\eta > \beta(\mu) = \mu &= \alpha_x^2 + \sum_{i=1}^{n_x} \alpha_{\tilde{W}_i}^2 \\
&= \left(\frac{\nu}{\lambda_{\min}(RP^{-1})} \right)^2 + \sum_{i=1}^{n_x} \left[\frac{w_i^*}{2\sqrt{\hat{q}_i}} + \sqrt{\frac{\nu}{2q_i\gamma_i}} \right]^2.
\end{aligned} \tag{62}
$$

To show ultimate boundedness of the closed-loop system (31), (51), and (52), assume[2] that $\tilde{\mathcal{D}}_\eta \subset \tilde{\mathcal{D}}_\alpha$ (see Fig. 5). Now, since $\dot{V}(e_x, e_z, \tilde{W}) \le 0$ for all $(e_x, e_z, \tilde{W}) \in \tilde{\mathcal{D}}_{\rm e} \backslash \tilde{\mathcal{D}}_{\rm r}$ and $\tilde{\mathcal{D}}_{\rm r} \subset \tilde{\mathcal{D}}_\alpha$, it follows that $\tilde{\mathcal{D}}_\alpha$ is positively invariant. Hence, if $(e_x(0), e_z(0), \tilde{W}(0)) \in \tilde{\mathcal{D}}_\alpha$, then it follows from Theorem 3.1 that the solution $(e_x(t), e_z(t), \hat{W}(t))$, $t \ge 0$, to (31), (51), and (52) is bounded with respect to $(e_x, \tilde{W})$ uniformly in $e_z(0)$ and, hence, ultimately bounded with respect to $(e_x, \tilde{W})$ uniformly in $e_z(0)$. To show that $\|P^{1/2}(x(t) - x_{\rm e})\| < \varepsilon$, $t \ge T$, note that $\tilde{\mathcal{D}}_\eta$ is also positively invariant and, hence, if there exists $t^* > 0$ such that $(e_x(t^*), e_z(t^*), \hat{W}(t^*)) \in \tilde{\mathcal{D}}_\eta$, then $(e_x(t^*), e_z(t^*), \hat{W}(t^*)) \in \tilde{\mathcal{D}}_\eta$, $t \ge t^*$. Alternatively, suppose the solution $(e_x(t), e_z(t), \hat{W}(t))$, $t \ge 0$, to (31), (51), and (52) remains in $\tilde{\mathcal{D}}_\alpha \backslash \tilde{\mathcal{D}}_\eta$. In this case, the Lyapunov-like function (53) is nonincreasing. Furthermore, it follows from (54) that (63), shown at the bottom of the next page, where

$$\dot{\delta}(x(t), z(t)) = \frac{\partial \delta}{\partial x}(x(t), z(t))\dot{x}(t) + \frac{\partial \delta}{\partial z}(x(t), z(t))\dot{z}(t) \tag{64}$$

$$\dot{\hat{\sigma}}(x(t), z(t)) = \frac{\partial \hat{\sigma}}{\partial x}(x(t), z(t))\dot{x}(t) + \frac{\partial \hat{\sigma}}{\partial z}(x(t), z(t))\dot{z}(t). \tag{65}$$

Note that since $\delta'(x, z)$ and $\hat{\sigma}'(x, z)$ are bounded and the state trajectory $(e_x(t), e_z(t), \hat{W}(t))$ is bounded, it follows from (31),

[2]This assumption is standard in the neural network literature and ensures that in the error space $\tilde{\mathcal{D}}_{\rm e}$ there exists at least one Lyapunov level set $\tilde{\mathcal{D}}_\eta \subset \tilde{\mathcal{D}}_\alpha$. In the case where the neural network approximation holds in $\mathbb{R}^{n_x} \times \mathbb{R}^{n_z}$, this assumption is automatically satisfied. See Remark 4.1 for further details.

$$
\begin{aligned}
&\ddot{V}(e_x(t), e_z(t), \tilde{W}(t)) \\
&= -2e_x^{\rm T}(t)R\dot{e}_x(t) + 2\dot{e}_x^{\rm T}(t)P\Big[B_{\rm u}[\delta(x(t), z(t)) - \delta(x_{\rm e}, z_{\rm e}) - G_n(x_{\rm e}, z_{\rm e})u_{\rm e} - \hat{W}^{\rm T}(t)\hat{\sigma}(x(t), z(t))]\Big] \\
&\quad + 2e_x^{\rm T}(t)P\Big[B_{\rm u}[\dot{\delta}(x(t), z(t)) - \dot{\hat{W}}^{\rm T}(t)\hat{\sigma}(x(t), z(t)) - \hat{W}^{\rm T}(t)\dot{\hat{\sigma}}(x(t), z(t))]\Big] \\
&\quad + \sum_{i=1}^{n_x} 2p_i b_i \dot{\hat{W}}_i^{\rm T}(t)\Big[e_{xi}(t)\hat{\sigma}_i(x(t), z(t)) - \gamma_i\|P^{1/2}e_x(t)\|\hat{W}_i(t)\Big] \\
&\quad + \sum_{i=1}^{n_x} 2p_i b_i \tilde{W}_i^{\rm T}(t)\Big[\dot{e}_{xi}(t)\hat{\sigma}_i(x(t), z(t)) + e_{xi}(t)\dot{\hat{\sigma}}_i(x(t), z(t)) - \gamma_i\Big(\frac{\rm d}{{\rm d}t}\|P^{1/2}e_x(t)\|\Big)\hat{W}_i(t) - \gamma_i\|P^{1/2}e_x(t)\|\dot{\hat{W}}_i(t)\Big], \ t \ge 0
\end{aligned} \tag{63}
$$

(51), (52) that $\dot{e}_x(t)$, $\dot{e}_z(t)$, $\dot{\hat{W}}(t)$ are also bounded and, hence, $\ddot{V}(e_x(t), e_z(t), \tilde{W}(t))$ is bounded. Thus, it follows from Barbalat's lemma [40, p. 192] that $\dot{V}(e_x(t), e_z(t), \tilde{W}(t)) \to 0$ as $t \to \infty$. Now, it follows from (55) that, since the quantity in the brackets in the right-hand side of (55) is strictly positive in $\tilde{\mathcal{D}}_\alpha \backslash \tilde{\mathcal{D}}_\eta$, $\|P^{1/2} e_x(t)\| \to 0$ as $t \to \infty$. Hence, in either case, there exists $T \geq 0$ such that $\|P^{1/2}(x(t) - x_{\rm e})\| < \varepsilon$, $t \geq T$, with $\varepsilon = \alpha^{-1}(\eta) = \sqrt{\eta}$ which yields (33).

Next, since (52) is input-to-state stable with e_x viewed as the input, it follows from Proposition 3.1 that the solution $e_z(t)$, $t \geq 0$, to (52) is ultimately bounded and, hence, the solution $(x(t), z(t), \hat{W}(t))$, $t \geq 0$, of the closed-loop system (16), (17), (30), and (31) is ultimately bounded for all $(x(0), z(0), \hat{W}(0)) \in \mathcal{D}_\alpha$. Furthermore, it follows from [41, Th. 1] that there exist a continuously differentiable, radially unbounded, positive–definite function $V_z : \mathbb{R}^{n_z} \to \mathbb{R}$ and class $\mathcal{K}$ functions $\gamma_1(\cdot), \gamma_2(\cdot)$ such that

$$V_z'(e_z)\tilde{f}_z(e_x, e_z) \leq -\gamma_1(\|e_z\|), \quad \|e_z\| \geq \gamma_2(\|P^{1/2} e_x\|). \tag{66}$$

Since the upper bound for $\|P^{1/2} e_x\|^2$ is given by α, it follows that the set given by:

$$\mathcal{D}_z \triangleq \left\{ z \in \mathbb{R}^{n_z} : V_z(z - z_{\rm e}) \leq \max_{\|z - z_{\rm e}\| = \gamma_2(\sqrt{\alpha})} V_z(z - z_{\rm e}) \right\} \tag{67}$$

is also positively invariant as long as[3] $\mathcal{D}_z \subset \mathcal{D}_{{\rm c}z}$. Now, since $\tilde{\mathcal{D}}_\alpha$ and $\mathcal{D}_z$ are positively invariant, it follows that:

$$\mathcal{D}_\alpha \triangleq \Big\{ (x, z, \hat{W}) \in \mathbb{R}^{n_x} \times \mathbb{R}^{n_z} \times \mathbb{R}^{s \times n_x} : (x - x_{\rm e}, z - z_{\rm e}, \hat{W} - W) \in \tilde{\mathcal{D}}_\alpha,\ z \in \mathcal{D}_z \Big\} \tag{68}$$

is also positively invariant.

Finally, to show that $x(t) \geq\geq 0$ and $z(t) \geq\geq 0$, $t \geq 0$, for all $(x_0, z_0) \in \overline{\mathbb{R}}_+^{n_x} \times \overline{\mathbb{R}}_+^{n_z}$ note that the closed-loop system (16), (30), and (31), is given by

$$\begin{aligned} \dot{x}(t) &= f_x(x(t), z(t)) + B_{\rm u} K(x(t) - x_{\rm e}) \\ &\quad - B_{\rm u} \hat{W}^{\rm T}(t) \hat{\sigma}(x(t), z(t)) \\ &= (A + B_{\rm u} K) x(t) + \Delta f(x(t), z(t)) \\ &\quad - B_{\rm u} \hat{W}^{\rm T}(t) \hat{\sigma}(x(t), z(t)) - B_{\rm u} K x_{\rm e} \\ &= \tilde{f}(t, x(t), z(t)) + v, \quad x(0) = x_0, \quad t \geq 0 \end{aligned} \tag{69}$$

where

$$\begin{aligned} \tilde{f}(t, x, z) &\triangleq (A + B_{\rm u} K) x + \Delta f(x, z) - B_{\rm u} \hat{W}^{\rm T}(t) \hat{\sigma}(x, z) \\ v &\triangleq -B_{\rm u} K x_{\rm e}. \end{aligned}$$

Since $\tilde{f}(t, x, z)$, $t \geq 0$, is essentially nonnegative with respect to x pointwise-in-time, $f_z(x, z)$ is essentially nonnegative with respect to z, and $v \geq\geq 0$, it follows from Proposition 2.1 that $x(t) \geq\geq 0$, $t \geq 0$, and $z(t) \geq\geq 0$, $t \geq 0$, for all $(x_0, z_0) \in \overline{\mathbb{R}}_+^{n_x} \times \overline{\mathbb{R}}_+^{n_z}$. □

To prove Theorem 5.1, first define $\hat{W}_{\rm u}^{\rm T}(t) \triangleq \text{block-diag}[\hat{W}_{{\rm u}1}^{\rm T}(t), \ldots, \hat{W}_{{\rm u}n_x}^{\rm T}(t)]$ and $K_{\rm u} \triangleq \text{diag}[k_{{\rm u}1}, \ldots, k_{{\rm u}n_x}]$, where

$$\hat{W}_{{\rm u}i}(t) = \begin{cases} 0, & \text{if } \hat{u}_i(t) < 0, \\ \hat{W}_i(t), & \text{otherwise}, \end{cases} \quad i = 1, \ldots, n_x, \tag{70}$$

$$k_{{\rm u}i} = \begin{cases} 0, & \text{if } \hat{u}_i(t) < 0, \\ k_i, & \text{otherwise}, \end{cases} \quad i = 1, \ldots, n_x. \tag{71}$$

[3]See Remark 4.1.

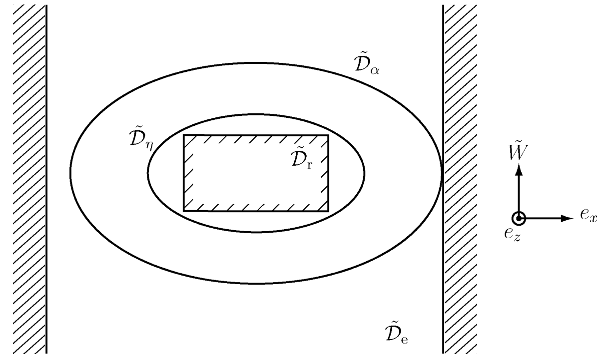

Fig. 5. Visualization of sets used in the proof of Theorem 4.1.

Next, note that with $u(t)$, $t \geq 0$, given by (36) it follows from (16), (19), and (29) that:

$$\begin{aligned} \dot{x}(t) &= Ax(t) + \Delta f(x(t), z(t)) + B_{\rm u}[K_{\rm u}(x(t) - x_{\rm e}) \\ &\quad - \hat{W}_{\rm u}^{\rm T}(t) \sigma(x(t), z(t))], \quad x(0) = x_0, \quad t \geq 0. \end{aligned} \tag{72}$$

Now, defining $e_x(t) \triangleq x(t) - x_{\rm e}$ and $e_z(t) \triangleq z(t) - z_{\rm e}$, and using (20)–(22), it follows from (17) and (72) that:

$$\begin{aligned} \dot{e}_x(t) &= Ae_x(t) + Ax_{\rm e} + \Delta f(x(t), z(t)) \\ &\quad + B_{\rm u}[K_{\rm u} e_x(t) - \hat{W}_{\rm u}^{\rm T}(t) \sigma(x(t), z(t))] \\ &= Ae_x(t) + B_{\rm u}[\delta(x(t), z(t)) - \delta(x_{\rm e}, z_{\rm e}) \\ &\quad - G_n(x_{\rm e}, z_{\rm e}) u_{\rm e} + K_{\rm u} e_x(t) - \hat{W}^{\rm T}(t) \sigma(x(t), z(t))] \\ &\quad + B_{\rm u}(\hat{W}(t) - \hat{W}_{\rm u}(t))^{\rm T} \sigma(x(t), z(t)), \\ e_x(0) &= x_0 - x_{\rm e}, \quad t \geq 0 \end{aligned} \tag{73}$$

and

$$\dot{e}_z(t) = \tilde{f}_z(e_x(t), e_z(t)), \quad e_z(0) = z_0 - z_{\rm e} \tag{74}$$

where $\tilde{f}_z(e_x, e_z) \triangleq f_z(e_x + x_{\rm e}, e_z + z_{\rm e}) - f_z(x_{\rm e}, z_{\rm e})$. Furthermore, since A is essentially nonnegative and asymptotically stable, it follows from [12, Th. 3.3] that there exist a positive *diagonal* matrix $P = \text{diag}[p_1, \ldots, p_{n_x}]$ and a positive–definite matrix $R \in \mathbb{R}^{n_x \times n_x}$ such that (39) holds.

Next, to show ultimate boundedness of the closed-loop system (38), (73), and (74) consider the Lyapunov-like function

$$V(e_x, e_z, \tilde{W}) = e_x^{\rm T} P e_x + \operatorname{tr} \tilde{W} Q^{-1} \tilde{W}^{\rm T} \tag{75}$$

where $Q \triangleq \text{diag}\left[\hat{q}_1, \ldots, \hat{q}_{n_x}\right] = \text{diag}\left[q_1/p_1 b_1, \ldots, q_{n_x}/p_{n_x} b_{n_x}\right]$ and $\tilde{W}(t) \triangleq \hat{W}(t) - W$ with $W^{\rm T}$ given by $W^{\rm T} = \text{block-diag}[W_1^{\rm T}, \ldots, W_{n_x}^{\rm T}]$. Note that (75) satisfies (9) with $x_1 = [e_x^{\rm T}, \tilde{W}_1^{\rm T} \ldots, \tilde{W}_{n_x}^{\rm T}]^{\rm T}$, $x_2 = e_z$, $\alpha(\|x_1\|) = \beta(\|x_1\|) = \|x_1\|^2$, where $\|x_1\|^2 \triangleq e_x^{\rm T} P e_x + \operatorname{tr} \tilde{W} Q^{-1} \tilde{W}^{\rm T}$. Furthermore, $\alpha(\|x_1\|)$ is a class $\mathcal{K}_\infty$ function. Now, letting $e_x(t)$, $t \geq 0$, denote the solution to (73) and using (24) and (38), it follows that the time derivative of $V(e_x, e_z, \tilde{W})$ along the closed-loop system trajectories is given by (76), shown at the bottom of the next page. Now, for each $i \in \{1, \ldots, n_x\}$ and for the two cases given in (70), the last term on the right-hand side of (76) gives:

1) If $\hat{u}_i(t) < 0$, then $\hat{W}_{ui}(t) = 0$ and, hence

$$\begin{aligned}2p_ib_i\Big(&e_{xi}(t)(\hat{W}_i(t)-\hat{W}_{\mathrm{u}i}(t))^{\mathrm{T}}\sigma_i(x(t),z(t))\\&-\gamma_i\|P^{1/2}e_x(t)\|\tilde{W}_i^{\mathrm{T}}(t)\hat{W}_i(t)\Big)\\=2p_ib_i\Big(&e_{xi}(t)(\tilde{W}_i(t)+W_i)^{\mathrm{T}}\sigma_i(x(t),z(t))\\&-\gamma_i\|P^{1/2}e_x(t)\|\|\tilde{W}_i(t)\|^2\\&-\gamma_i\|P^{1/2}e_x(t)\|\tilde{W}_i^{\mathrm{T}}(t)W_i\Big)\\\le 2b_i&\sqrt{p_i\hat{q}_is_i}\|P^{1/2}e_x(t)\|\|\hat{q}_i^{-1/2}\tilde{W}_i(t)\|\\&+2b_i\sqrt{p_is_i}w_i^*\|P^{1/2}e_x(t)\|\\&-2p_i\hat{q}_ib_i\gamma_i\|P^{1/2}e_x(t)\|\|\hat{q}_i^{-1/2}\tilde{W}_i(t)\|^2\\&+2p_ib_i\hat{q}_i^{1/2}\gamma_iw_i^*\|P^{1/2}e_x(t)\|\|\hat{q}_i^{-1/2}\tilde{W}_i(t)\|.\end{aligned}$$

2) Otherwise, $\hat{W}_{ui}(t) = \hat{W}_i(t)$ and, hence

$$\begin{aligned}2p_ib_i\Big(&e_{xi}(t)(\hat{W}_i(t)-\hat{W}_{\mathrm{u}i}(t))^{\mathrm{T}}\sigma_i(x(t),z(t))\\&-\gamma_i\|P^{1/2}e_x(t)\|\tilde{W}_i^{\mathrm{T}}(t)\hat{W}_i(t)\Big)\\=&-2p_ib_i\gamma_i\|P^{1/2}e_x(t)\|\tilde{W}_i^{\mathrm{T}}(t)\hat{W}_i(t)\\\le&-2p_ib_i\gamma_i\|P^{1/2}e_x(t)\|\tilde{W}_i^{\mathrm{T}}(t)\tilde{W}_i(t)\\&+2p_ib_i\hat{q}_i^{1/2}\gamma_iw_i^*\|P^{1/2}e_x(t)\|\|\hat{q}_i^{-1/2}\tilde{W}_i(t)\|\\\le& 2\sqrt{q_ib_is_i}\|P^{1/2}e_x(t)\|\|\hat{q}_i^{-1/2}\tilde{W}_i(t)\|\\&+2b_i\sqrt{p_is_i}w_i^*\|P^{1/2}e_x(t)\|\\&-2q_i\gamma_i\|P^{1/2}e_x(t)\|\|\hat{q}_i^{-1/2}\tilde{W}_i(t)\|^2\\&+2q_i\hat{q}_i^{-1/2}\gamma_iw_i^*\|P^{1/2}e_x(t)\|\|\hat{q}_i^{-1/2}\tilde{W}_i(t)\|.\end{aligned}$$

Hence, it follows from (76) that in either case (77), shown at the top of the next page. Next, completing squares yields

$$\begin{aligned}\dot{V}&(e_x(t),e_z(t),\tilde{W}(t))\\&\le-\lambda_{\min}(RP^{-1})\|P^{1/2}e_x(t)\|^2\\&\quad+2\|P^{1/2}e_x(t)\|\left(\sum_{i=1}^{n_x}p_ib_i^2\varepsilon_i^{*2}\right)^{1/2}\\&\quad+\sum_{i=1}^{n_x}2\sqrt{q_ib_is_i}\|P^{1/2}e_x(t)\|\|\hat{q}_i^{-1/2}\tilde{W}_i(t)\|\\&\quad+\sum_{i=1}^{n_x}2b_i\sqrt{p_is_i}w_i^*\|P^{1/2}e_x(t)\|\\&\quad-2\sum_{i=1}^{n_x}q_i\gamma_i\|P^{1/2}e_x(t)\|\|\hat{q}_i^{-1/2}\tilde{W}_i(t)\|^2\\&\quad+2\sum_{i=1}^{n_x}q_i\hat{q}_i^{-1/2}\gamma_iw_i^*\|P^{1/2}e_x(t)\|\|\hat{q}_i^{-1/2}\tilde{W}_i(t)\|\\&=\|P^{1/2}e_x(t)\|\Bigg[-\lambda_{\min}(RP^{-1})\|P^{1/2}e_x(t)\|\\&\quad+2\left(\sum_{i=1}^{n_x}p_ib_i^2\varepsilon_i^{*2}\right)^{1/2}\\&\quad+\sum_{i=1}^{n_x}2b_i\sqrt{p_is_i}w_i^*-\sum_{i=1}^{n_x}2q_i\gamma_i\|\hat{q}_i^{-1/2}\tilde{W}_i(t)\|^2\\&\quad+\sum_{i=1}^{n_x}2q_i\gamma_i\left(\sqrt{\frac{b_is_i}{q_i\gamma_i^2}}+\frac{w_i^*}{\sqrt{\hat{q}_i}}\right)\|\hat{q}_i^{-1/2}\tilde{W}_i(t)\|\Bigg]\\&=\|P^{1/2}e_x(t)\|\Bigg[-\lambda_{\min}(RP^{-1})\|P^{1/2}e_x(t)\|+\nu\\&\quad-\sum_{i=1}^{n_x}2q_i\gamma_i\left[\|\hat{q}_i^{-1/2}\tilde{W}_i(t)\|-\frac{1}{2}\left(\sqrt{\frac{b_is_i}{q_i\gamma_i^2}}+\frac{w_i^*}{\sqrt{\hat{q}_i}}\right)\right]^2\Bigg]\end{aligned}\tag{78}$$

where

$$\begin{aligned}\nu\triangleq&\left(\sum_{i=1}^{n_x}p_ib_i^2\varepsilon_i^{*2}\right)^{1/2}\\&+\sum_{i=1}^{n_x}\left[2b_i\sqrt{p_is_i}w_i^*+\frac{q_i\gamma_i}{2}\left(\sqrt{\frac{b_is_i}{q_i\gamma_i^2}}+\frac{w_i^*}{\sqrt{\hat{q}_i}}\right)^2\right]\end{aligned}$$

and $\hat{q}_i = q_i/p_ib_i$. Now, for

$$\|P^{1/2}e_x\|\ge\frac{\nu}{\lambda_{\min}(RP^{-1})}\triangleq\alpha_x\tag{79}$$

$$\begin{aligned}\dot{V}(e_x(t),e_z(t),\tilde{W}(t))=&\,2e_x^{\mathrm{T}}(t)P\Big[Ae_x(t)+B_{\mathrm{u}}[\delta(x(t),z(t))-\delta(x_{\mathrm{e}},z_{\mathrm{e}})-G_n(x_{\mathrm{e}},z_{\mathrm{e}})u_{\mathrm{e}}+K_{\mathrm{u}}e_x(t)-\hat{W}^{\mathrm{T}}(t)\sigma(x(t),z(t))]\\&+B_{\mathrm{u}}(\hat{W}(t)-\hat{W}_{\mathrm{u}}(t))^{\mathrm{T}}\sigma(x(t),z(t))\Big]+2\mathrm{tr}\,\tilde{W}^{\mathrm{T}}(t)Q^{-1}\dot{\hat{W}}(t)\\=&-e_x^{\mathrm{T}}(t)Re_x(t)+2e_x^{\mathrm{T}}(t)PB_{\mathrm{u}}K_{\mathrm{u}}e_x(t)+\sum_{i=1}^{n_x}2p_ib_ie_{xi}(t)\Big[-\tilde{W}_i^{\mathrm{T}}(t)\sigma_i(x(t),z(t))+\varepsilon_i(x(t),z(t))\Big]\\&+\sum_{i=1}^{n_x}2p_ib_ie_{xi}(t)(\hat{W}_i(t)-\hat{W}_{\mathrm{u}i}(t))^{\mathrm{T}}\sigma_i(x(t),z(t))\\&+\sum_{i=1}^{n_x}2p_ib_i\tilde{W}_i^{\mathrm{T}}(t)\Big[e_{xi}(t)\sigma_i(x(t),z(t))-\gamma_i\|P^{1/2}(x(t)-x_{\mathrm{e}})\|\hat{W}_i(t)\Big]\\\le&-e_x^{\mathrm{T}}(t)Re_x(t)+\sum_{i=1}^{n_x}2p_ib_ie_{xi}(t)\varepsilon_i(x(t),z(t))\\&+\sum_{i=1}^{n_x}2p_ib_i\Big(e_{xi}(t)(\hat{W}_i(t)-\hat{W}_{\mathrm{u}i}(t))^{\mathrm{T}}\sigma_i(x(t),z(t))-\gamma_i\|P^{1/2}e_x(t)\|\tilde{W}_i^{\mathrm{T}}(t)\hat{W}_i(t)\Big).\end{aligned}\tag{76}$$

$$\begin{aligned}\dot{V}(e_x(t),e_z(t),\tilde{W}(t)) \leq &- e_x^{\mathrm{T}}(t)Re_x(t) + \sum_{i=1}^{n_x} 2p_i b_i e_{xi}(t)\varepsilon_i(x(t),z(t)) \\ &+ \sum_{i=1}^{n_x}\Big(2\sqrt{q_i b_i s_i}\|P^{1/2}e_x(t)\|\|\hat{q}_i^{-1/2}\tilde{W}_i(t)\| + 2b_i\sqrt{p_i s_i}w_i^*\|P^{1/2}e_x(t)\| \\ &- 2q_i\gamma_i\|P^{1/2}e_x(t)\|\|\hat{q}_i^{-1/2}\tilde{W}_i(t)\|^2 + 2q_i\hat{q}_i^{-1/2}\gamma_i w_i^*\|P^{1/2}e_x(t)\|\|\hat{q}_i^{-1/2}\tilde{W}_i(t)\|\Big) \\ \leq &- e_x^{\mathrm{T}}(t)Re_x(t) + 2\|P^{1/2}e_x(t)\|\left(\sum_{i=1}^{n_x} p_i b_i^2 \varepsilon_i^{*2}\right)^{1/2} \\ &+ \sum_{i=1}^{n_x} 2\sqrt{q_i b_i s_i}\|P^{1/2}e_x(t)\|\|\hat{q}_i^{-1/2}\tilde{W}_i(t)\| + \sum_{i=1}^{n_x} 2b_i\sqrt{p_i s_i}w_i^*\|P^{1/2}e_x(t)\| \\ &- 2\sum_{i=1}^{n_x} q_i\gamma_i\|P^{1/2}e_x(t)\|\|\hat{q}_i^{-1/2}\tilde{W}_i(t)\|^2 + 2\sum_{i=1}^{n_x} q_i\hat{q}_i^{-1/2}\gamma_i w_i^*\|P^{1/2}e_x(t)\|\|\hat{q}_i^{-1/2}\tilde{W}_i(t)\|.\end{aligned} \tag{77}$$

or

$$\|\hat{q}_i^{-1/2}\tilde{W}_i\| \geq \frac{1}{2}\left(\frac{\sqrt{b_i s_i}}{q_i\gamma_i^2} + \frac{w_i^*}{\hat{q}_i}\right) + \sqrt{\frac{\nu}{2q_i\gamma_i}} \triangleq \alpha_{\tilde{W}_i}, \quad i = 1,\ldots,n_x \tag{80}$$

it follows that $\dot{V}(e_x(t),e_z(t),\tilde{W}(t)) \leq 0$ for all $t \geq 0$; that is, $\dot{V}(e_x(t),e_z(t),\tilde{W}(t)) \leq 0$ for all $(e_x(t),e_z(t),\tilde{W}(t)) \in \tilde{\mathcal{D}}_{\mathrm{e}}\backslash\tilde{\mathcal{D}}_{\mathrm{r}}$ and $t \geq 0$, where $\tilde{\mathcal{D}}_{\mathrm{e}}$ and $\tilde{\mathcal{D}}_{\mathrm{r}}$ are given by (58) and (59), respectively. Now, the proof follows as in the proof of Theorem 4.1. □

Acknowledgment

The authors would like to thank Dr. E. Lavretsky for several stimulating discussions on approximation theory of essentially nonnegative functions.

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